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Level 5
Exemplar 1

2024-DSE
MATH EP
M1

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HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2024

Candidate Number									

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8:30 am – 11:00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of TWO sections, A and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

(7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

A031E001

SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where a and b are constants such that $6 < b < 15$.

x	0	3	6	~~b~~ 8	15
$\mathrm{P}(X = x)$	0.3	~~a~~ 0.2	0.1	0.2	0.2

It is given that $\mathrm{Var}(5X) = 739$.

(a) Find a and b.

(b) Let C be the event that $0 < X \le 7$.

(i) Let D be the event that $4 < X \le 15$. Are C and D independent? Explain your answer.

(ii) Let E be an event such that $\mathrm{P}(E) \ne 0$. If C and E are mutually exclusive, write down the greatest possible value of $\mathrm{P}(E)$.

(7 marks)

a) $0.3 + a + 0.1 + 0.2 + 0.2 = 1$

$a = 0.2$

$E(X) = 0(0.3) + 3(0.2) + 6(0.1) + b(0.2) + 15(0.2)$

$= 4.2 + 0.2b$

$\mathrm{Var}(X) = 0^2(0.3) + 3^2(0.2) + 6^2(0.1) + b^2(0.2) + 15^2(0.2) - (4.2 + 0.2b)^2$

$= 50.4 + 0.2b^2 - 17.64 - 1.68b - 0.04b$

$= 0.16b^2 - 1.68b + 32.76$

$\mathrm{Var}(5X) = 739$

$5^2\,\mathrm{Var}(X) = 739$

$0.16b^2 - 1.68b + 32.76 = 739 \div 25$

$0.16b^2 - 1.68b + 32.76 = 29.56$

$0.16b^2 - 1.68b + 3.2 = 0$

$b = 8$ or $b = 2.5$ (rej)

Answers written in the margins will not be marked.

b i) $P(C) = 0.2+0.1 = 0.3$

$P(D) = 0.1+0.2+0.2 = 0.5$

$P(C \cap D) = 0.1$

$\because P(C) \times P(D)$

$= 0.3 \times 0.5$

$= 0.15$

$\neq P(D)$ (0.1)

$\therefore$ C and D are not independent

b ii) $\because$ C and E are mutually exclusive.

$\therefore P(C \cap E) = 0$

possible range $= 7 < X \leq 15$

$\therefore$ greatest possible value of $P(E)$

$= 0.2 + 0.2$

$= 0.4$

Answers written in the margins will not be marked.

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2. In an orchestra, $\frac{3}{5}$ of the members wear glasses. Among the male members, $\frac{4}{9}$ of them wear glasses. The probability that a randomly selected member is a female not wearing glasses is $\frac{3}{20}$.

(a) Given that a randomly selected member does not wear glasses, find the probability that the member is a female.

(b) Find the probability that a randomly selected member is a female wearing glasses.

(6 marks)

a) P(female | does not wear glasses)

$= \dfrac{\frac{3}{20}}{1-\frac{3}{5}}$

$= \frac{3}{8}$

b) P(required)

$= \frac{3}{5} - \frac{4}{9}$

$= \frac{7}{45}$

Answers written in the margins will not be marked.

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Answers written in the margins will not be marked.

3. When a coin is tossed, the probability of getting a tail is p, where $0 < p < 1$. When the coin is tossed 20 times, the ratio of the probability of getting 1 tail to the probability of getting 3 tails is $49 : 57$.

(a) Find p.

(b) The coin is tossed k times. Find the least value of k so that the probability of getting at least 1 tail is greater than 0.85.

(6 marks)

a) $C_1^{20}(p)(1-p)^{19} : C_3^{20}(p)^3(1-p)^{17} = 49:57$

$$\frac{C_1^{20}(p)(1-p)^{19}}{C_3^{20}(p)^3(1-p)^{17}} = \frac{49}{57}$$

$$\frac{(1-p)^2}{57p^2} = \frac{49}{57}$$

$1-2p+p^2 = 49p^2$

$48p^2+2p-1=0$

$p=\frac{1}{8}$ or $p=\frac{-1}{6}$ (rej)

b) $1-(1-\frac{1}{8})^k > 0.85$

$(\frac{7}{8})^k < 0.15$

$k \ln\frac{7}{8} < \ln 0.15$

$k > 14.20729573$

$\therefore$ least value of $k = 15$

4. The weekly revision time of each student in a school follows a normal distribution with a mean of μ hours. A random sample of 81 students is drawn from the school. The mean and the standard deviation of the weekly revision time of these students are 13 hours and 1.75 hours respectively.

(a) The width of a $\beta\%$ confidence interval for μ is 0.7. Find β.

(b) It is given that there are 36 boys in the sample, and the mean and the standard deviation of the weekly revision time of these boys are 12.5 hours and 2 hours respectively. Find the standard deviation of the weekly revision time of the girls in the sample.

[Hint: The sample standard deviation is $\sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}$.]

(6 marks)

a) let $\overline{X}_{81} \sim N\left(13, \left(\frac{1.75}{\sqrt{81}}\right)^2\right)$, z be standard score.

$2 \cdot z \frac{1.75}{\sqrt{81}} = 0.7$

$z = 1.8$

$\beta = 46.41 \times 2$

$= 92.82$

$\therefore \beta = 93$

b) no. of girls $= 81 - 36 = 45$

Sample mean for girls $= \frac{81 \times 13 - 36 \times 12.5}{45} = 13.4$ hours

Boys: $\sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)} = 2$

$\sqrt{\frac{1}{36-1}\left(\sum_{i=1}^{n} x_i^2 - 36(12.5)^2\right)} = 2$

$\sum_{i=1}^{n} x_i^2 = 5765$

$\therefore$ sample s.d. of girls $= \sqrt{\frac{1}{45-1}\left(\sum_{i=1}^{n} x_i^2 - 45(13.4)^2\right)}$

$=$

Answers written in the margins will not be marked.

5. Let n be a positive number.

(a) Expand $\frac{2}{e^{nx}}$ in ascending powers of x as far as the term in x^3 .

(b) Consider the expansion of $(1+4x)^m + \frac{2}{e^{nx}}$, where m is a positive integer. The coefficients of x and x^2 in the expansion are 24 and 980 respectively. Find the coefficient of x^3 in the expansion.

(7 marks)

a) $\frac{2}{e^{nx}}$

$= 2e^{-nx}$

$= 2\left(1 - nx + \frac{(-nx)^2}{2!} + \frac{(-nx)^3}{3!} + \cdots\right)$

$= 2\left(1 - nx + \frac{n^2}{2}x^2 - \frac{n^3}{6}x^3 + \cdots\right)$

$= 2 - 2nx + n^2x^2 - \frac{n^3}{3}x^3 + \cdots$

b) $(1+4x)^m + \frac{2}{e^{nx}}$

$= (1+4x)^m + 2e^{-nx}$

$= 1 + 4mx + \frac{m(m-1)}{2}(4x)^2 + \frac{m(m-1)(m-2)}{3!}(4x)^3 + \cdots + 2 - 2nx + n^2x^2 - \frac{n^3}{3}x^3 + \cdots$

coeff of $x = 24$

$4m - 2n = 24$

$2m - n = 12$

$n = 2m - 12$ — ①

coeff of $x^2 = 980$

$\frac{m(m-1)}{2}(4)^2 + n^2 = 980$

$8m(m-1) + n^2 = 980$ — ②

sub ① into ②,

$8m(m-1) + (2m-12)^2 = 980$

$8m^2 - 8m + 4m^2 - 48m + 144 = 980$

$12m^2 - 56m - 836 = 0$

$m = 11$ or $m = \frac{-19}{3}$ (rej)

$n = 10$

$\therefore$ coeff of $x^3 = \frac{m(m-1)(m-2)}{3!}(4)^3 - \frac{n^3}{3}$

$= \frac{11(10)(9)}{6}(4)^3 - \frac{(10)^3}{3} = \frac{30680}{3}$

Answers written in the margins will not be marked.

6. (a) Let $e^u = (x^2 + x + e)^{2x+1}$.

(i) Express u in the form of $\mathrm{p}(x)\ln(\mathrm{q}(x))$, where $\mathrm{p}(x)$ and $\mathrm{q}(x)$ are polynomials.

(ii) Express $\frac{\mathrm{d}}{\mathrm{d}x}e^u$ in terms of x .

(b) The equation of the curve Γ is $y = (x^2 + x + e)^{2x+1}$. Denote the point of intersection of Γ and the y-axis by H . Find the equation of the tangent to Γ at H .

$(0,e)$

(7 marks)

ai) $e^u = (x^2+x+e)^{2x+1}$

$u = (2x+1)\ln(x^2+x+e)$

aii) $e^u = (x^2+x+e)^{2x+1}$

$u = (2x+1)\ln(x^2+x+e)$

$\frac{d}{dx} = 2\ln(x^2+x+e) + (2x+1)\frac{1}{2x+1+e}$

$= 2\ln(x^2+x+e) + \frac{2x+1}{2x+1+e}$

$\therefore \frac{d}{dx}e^u = 2\ln(x^2+x+e) + \frac{2x+1}{2x+1+e}$

b) $y = (x^2+x+e)^{2x+1}$

For y-int , sub $x=0$

$y = (e)^1$

$y = e$

$\therefore$ y-int $= e$

$\therefore H = (0,e)$

Slope $= \frac{d}{dx}\Big|_{(0,e)} = 2\ln e + \frac{1}{1+e} = 2 + \frac{1}{1+e} = \frac{3+2e}{1+e}$

$\therefore$ Equation of tangent :

$\frac{y-e}{x-0} = \frac{3+2e}{1+e}$

$(1+e)y - e - e^2 = (3+2e)x$

$(3+2e)x - (1+e)y + (e^2+e) = 0$

Answers written in the margins will not be marked.

7. A computer programme adjusts the length and the breadth of a rectangular digital picture, such that the length of its diagonal remains constant while its breadth decreases at the constant rate of 0.5 cm s^{-1}. Initially, the length and the breadth of the picture are 20 cm and 15 cm respectively. Denote the breadth of the picture by x cm. Find the rate of change of the area of the picture when $x = 7$. (4 marks)

let l cm be length, b cm be breadth, d cm be diagonal length

A cm² be area

$$\frac{db}{dt} = -0.5 \text{ cms}^{-1}$$

$$A = l \cdot b$$

diff both sides w.r.t. t

$$\frac{dA}{dt} = \frac{dl}{dt} b + l \cdot \frac{db}{dt}$$

sub $\frac{dl}{dt} = 0.375$, $b = 7$,

$l = 24$, $\frac{db}{dt} = -0.5$

$$\frac{dA}{dt} = 0.375(7) + 24(-0.5)$$

$$= -9.375 \text{ cm}^2\text{s}^{-1}$$

~~$l^2 + b^2 = d^2$~~

~~sub $l = 20$, $b = 15$~~

~~$20^2 + 15^2 = d^2$~~

~~$d = 25$~~

$$l^2 + b^2 = d^2$$

diff both sides w.r.t. t

$$\frac{dl}{dt}(2l) + \frac{db}{dt}(2b) = \frac{dd}{dt}(2d)$$

$$\frac{dl}{dt}(2l) = -\frac{db}{dt}(2b)$$

sub $l = 20$, $b = 15$, $\frac{db}{dt} = -0.5$

$$\frac{dl}{dt}(2 \times 20) = 0.5(2 \times 15)$$

$$\frac{dl}{dt} = 0.375 \text{ cms}^{-1}$$

$$l^2 + b^2 = d^2$$

sub $l = 20$, $b = 15$

$$d = \sqrt{20^2 + 15^2}$$

$$d = 25$$

$$l^2 + b^2 = d^2$$

sub $b = 7$, $d = 25$

$$l = \sqrt{25^2 - 7^2}$$

$$l = 24$$

Answers written in the margins will not be marked.

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8. (a) Using the Standard Normal Distribution Table on page 24, evaluate $\int_0^{0.5} e^{\frac{-x^2}{2}} dx$.

(b) Consider the curve $C: y=(2x-1)e^{\frac{-x^2}{2}}$, where $x \geq 0$. Using the result of (a), find the area of the region bounded by C, the x-axis and the y-axis.

(7 marks)

a) $\int_0^{0.5} e^{-\frac{x^2}{2}} dx$

$= \sqrt{2\pi} \int_0^{0.5} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx$

$\approx \sqrt{2\pi}\ (0.1915)$

$= 0.4800$

b) $C: y = (2x-1)e^{-\frac{x^2}{2}}$

For x-int, sub $y=0$

$(2x-1)e^{-\frac{x^2}{2}} = 0$

$x = 0.5$ or $e^{-\frac{x^2}{2}} = 0$ (rej)

$\therefore$ x-int $= 0.5$

$\therefore$ Area $= \int_0^{0.5} (2x-1)e^{-\frac{x^2}{2}} dx$

$= \int_0^{0.5} (2xe^{-\frac{x^2}{2}} - e^{-\frac{x^2}{2}}) dx$

$= \int_0^{0.5} (2xe^{-\frac{x^2}{2}}) dx - \int_0^{0.5} e^{-\frac{x^2}{2}} dx$

let $u = -\frac{x^2}{2}$, when $x = 0.5$, $u = -\frac{1}{8}$

$du = -x$ $\quad x=0$, $u = 1$

$= -\int_1^{-\frac{1}{8}} e^u du - 0.4800$

$= \int_{-\frac{1}{8}} e^u du - 0.4800$

$= [e^u]_{-\frac{1}{8}} - 0.4800$

$= 1.3558$ sq. units

Answers written in the margins will not be marked.

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SECTION B (50 marks)

9. The weight of each pumpkin in a large market follows a normal distribution with a mean of μ kg and a standard deviation of σ kg. It is given that 30.85% of the pumpkins in the market each weighs more than 5.7 kg while 78.88% of the pumpkins each weighs between $(\mu - 1.5)$ kg and $(\mu + 1.5)$ kg.

(a) Find μ and σ. (3 marks)

(b) Suppose that 16 pumpkins are randomly chosen in the market. Find the probability that the mean weight of these pumpkins does not exceed 5.4 kg. (2 marks)

(c) The following table shows the grades and the prices of the pumpkins in the market.

Weight of a pumpkin (W kg)	$W \le 3.6$	$3.6 < W \le 5.7$	$W > 5.7$
Grade	C	B	A
Price ($)	50	80	100

Suppose that 8 pumpkins are randomly chosen in the market and these pumpkins are put into a trolley.

(i) Find the expected price of the pumpkins in the trolley.

(ii) Find the probability that there are at least 5 grade B pumpkins and at least 1 grade A pumpkin in the trolley.

(6 marks)

a) let $X \sim N(\mu, \sigma^2)$

$P(X > 5.7) = 0.3085$

$P(Z > \frac{5.7-\mu}{\sigma}) = 0.3085$

$\frac{5.7-\mu}{\sigma} = 0.5$

$\therefore \frac{5.7-\mu}{1.2} = 0.5$

$\mu = 5.1$

$P(\mu-1.5 \le X \le \mu+1.5) = 0.7888$

$P(\frac{-1.5}{\sigma} \le Z \le \frac{1.5}{\sigma}) = 0.7888$

$P(0 \le Z \le \frac{1.5}{\sigma}) = 0.3944$

$\frac{1.5}{\sigma} = 1.25$

$\sigma = 1.2$

b) let $\bar{X}_{16} \sim N(5.1, (\frac{1.2}{\sqrt{16}})^2)$

$P(X \le 5.4)$

$= P(Z \le \frac{5.4-5.1}{\frac{1.2}{\sqrt{16}}})$

$= P(Z \le 1)$

$= 0.8413$

Answers written in the margins will not be marked.

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Answers written in the margins will not be marked.

(i) $W \sim N(5.1, 1.2^2)$

$P(W \le 3.6)$ | $P(W > 5.7)$ | $P(3.6 < W \le 5.7)$

$= P\left(Z \le \frac{3.6-5.1}{1.2}\right)$ | $= P\left(Z > \frac{5.7-5.1}{1.2}\right)$ | $= 1 - 0.1056 - 0.3085$

$= P(Z \le -1.25)$ | $= P(Z > 0.5)$ | $= 0.5859$

$= 0.1056$ | $= 0.3085$

Expected price

$= (0.1056 \times 50 + 0.5859 \times 80 + 0.3085 \times 100) \times 8$

$= \$664.0160$

(ii) P(required)

$= C^8_2 (0.1056)^2 \times C^6_5 (0.5859)^5 \times (0.3085) + C^8_1 (0.1056) \times C^7_5 (0.5859)^5 \times (0.3085)^2$

$+ C^8_6 (0.5859)^6 \times (0.3085)^2 + C^8_1 (0.1056) \times C^7_6 (0.5859)^6 \times (0.3085)$

$+ C^8_7 (0.5859)^7 \times (0.3085) + C^8_5 (0.5859)^5 (0.3085)^3$

$= 0.5101$

Answers written in the margins will not be marked.

10. A courier delivers goods every day. The number of delays in delivery on a day follows a Poisson distribution with a mean of 1.6 . A day is regarded as *smooth* if there are fewer than 3 delays on that day.

(a) Find the probability that a certain day is *smooth*. (2 marks)

(b) Find the probability that all the 7 days in a certain week are *smooth*. (2 marks)

(c) Given that all the 7 days in a certain week are *smooth*, find the probability that there are exactly 10 delays in that week. (4 marks)

(d) Given that there are no delays in at least 2 days in a certain week, find the probability that all the 7 days in that week are *smooth*. (4 marks)

a) P(smooth)

$= e^{-1.6}\left(\frac{1.6^0}{0!} + \frac{1.6^1}{1!} + \frac{1.6^2}{2!}\right)$

$= 0.7834$

b) P(required)

$= (0.7834)^7$

$= 0.1810$

~~c) P(10 delays | 7 days in a week are smooth)~~

~~0.1810~~

c) $P(0 \text{ delays}) = \frac{e^{-1.6}1.6^0}{0!} = e^{-1.6}$

$P(1 \text{ delay}) = \frac{e^{-1.6}1.6^1}{1!} = 1.6e^{-1.6}$

$P(2 \text{ delays}) = \frac{e^{-1.6}1.6^2}{2!} = 1.28e^{-1.6}$

P(10 delays | 7 days in a week are smooth)

$$= \frac{C^7_4(1.6e^{-1.6})^4(1.28e^{-1.6})^3 + C^7_1(e^{-1.6})C^6_2(1.6e^{-1.6})^2(1.28e^{-1.6})^4 + C^7_2(e^{-1.6})^2(1.28e^{-1.6})^5}{0.1810}$$

$= 0.0963$

Answers written in the margins will not be marked.

d) P(7 days in a week are smooth | no delays in at least 2 days)

$$= \frac{C^7_2 \, (e^{-1.6})^2 \, (1.28e^{-1.6})^5}{1-(1-e^{-1.6})^7 - C^7_1 (e^{-1.6})(1-e^{-1.6})^6}$$

$= 0.0023$

11. The accumulative rainfall of city M on a certain day increases at a rate of P mm per hour. It is given that

$$P = a(-t^2 + 10t + 8)e^{bt},$$

where a and b are constants, and t $(0 \le t \le 4)$ is the number of hours elapsed since 7 am on that day. It is found that $\ln\left(\dfrac{P}{-t^2+10t+8}\right)$ is a linear function of t, and the graph of this linear function passes through the point $(3, -0.1)$ and the intercept on the horizontal axis is 2.5.

x-axis (2.5, 0)

(a) Express $\ln\left(\dfrac{P}{-t^2+10t+8}\right)$ as a linear function of t. (1 mark)

(b) Find the exact values of a and b. (3 marks)

(c) Using the trapezoidal rule with 4 sub-intervals, estimate the accumulative rainfall of city M from 7 am to 11 am on that day. (2 marks)

(d) The accumulative rainfall of city N on the same day increases at a rate of Q mm per hour. It is given that

$$Q = \frac{16(2t+5)e^{0.4t}}{4te^{0.4t}+3},$$

where t $(0 \le t \le 4)$ is the number of hours elapsed since 7 am on that day.

(i) Find $\int Q\,\mathrm{d}t$.

(ii) Someone claims that the sum of the accumulative rainfalls of city M and city N from 7 am to 11 am on that day is greater than 160 mm. Do you agree? Explain your answer.

(8 marks)

a) $P = a(-t^2+10t+8)e^{bt}$

$\dfrac{P}{-t^2+10t+8} = ae^{bt}$

$\ln\left(\dfrac{P}{-t^2+10t+8}\right) = bt + \ln a$, which is a linear function of t.

b) slope $= b$

$b = \dfrac{0+0.1}{2.5-3}$

$b = -0.2$

sub $(2.5, 0)$

$0 = 2.5(-0.2) + \ln a$

$\ln a = 0.5$

$a = e^{0.5}$

Answers written in the margins will not be marked.

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← let $f(t) = e^{0.5}(-t^2+10t+8)e^{-0.2t}$

c) Accumulative rainfall

$= \int_0^4 e^{0.5}(-t^2+10t+8)e^{-0.2t}\,dt$ $\quad \Delta t = \frac{4-0}{4} = 1$

$\approx \frac{1}{2}\left\{f(0)+2\left[f(1)+f(2)+f(3)\right]+f(4)\right\}$

← ≈ 94.1599635

$= 94.1600$ mm

d i) $\int Q\,dt$

$= \int \frac{16(2t+5)e^{0.4t}}{4te^{0.4t}+3}\,dt$

$= \int \frac{16(2t+5)e^{0.4t}}{4te^{0.4t}+3} \cdot \frac{1}{4e^{0.4t}+1.6te^{0.4t}}\,d(4te^{0.4t}+3)$

$= 20\int \left(\frac{1}{4te^{0.4t}+3}\right)d(4te^{0.4t}+3)$

$= 20\ln\left|4te^{0.4t}+3\right| + C$

when $t=0$, $\int Q\,dt = 0$

$C = -20\ln 3$

$\therefore \int Q\,dt = 20\ln\left|4te^{0.4t}+3\right| - 20\ln 3$

$= 20\ln\left|\frac{4te^{0.4t}+3}{3}\right|$

d ii) $\int_0^4 Q\,dt$

$= \left[20\ln\left|4te^{0.4t}+3\right|\right]_0^4$ (from (d i))

$= 66.22266184$ mm

$\therefore$ Accumulative rainfall

$\approx 94.1599635 + 66.22266184$

$= 160.3826$ mm

$f(t) = e^{0.5-0.2t}(-t^2+10t+8)$

$f'(t) = -0.2e^{0.5-0.2t}(-t^2+10t+8) + e^{0.5-0.2t}(-2t+10)$

$= e^{0.5-0.2t}(0.2t^2-2t-1.6-2t+10)$

$= e^{0.5-0.2t}(0.2t^2-4t+8.4)$

$f''(t) = -0.2e^{0.5-0.2t}(0.2t^2-4t+8.4) + e^{0.5-0.2t}(0.4t-4)$

$= e^{0.5-0.2t}(-0.04t^2+0.8t-1.68+0.4t-4)$

$= e^{0.5-0.2t}(-0.04t^2+1.2t-5.68) < 0$ when $0 \le t \le 4$

$\therefore$ ans in (c) is an underestimation

$\therefore$ accumulative rainfall (160.3826 mm) is an underestimation

$\therefore$ greater than 160 mm $\therefore$ I agree.

Answers written in the margins will not be marked.

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12. Let R be the total revenue (in thousand dollars) of an online shop. It is given that

$$\frac{dR}{dt}=\frac{2e^{0.5t}-5e^{-0.5t}}{2e^{0.5t}+5e^{-0.5t}-5}+2 \ ,$$

where t $(t \geq 0)$ is the number of months elapsed since the shop opens.

(a) Does the greatest rate of change of the total revenue of the shop exceed 4 thousand dollars per month? Explain your answer. (4 marks)

(b) Let P be the total profit (in thousand dollars) of the shop. It is given that

$$\frac{dP}{dt}=\frac{dR}{dt}-10(0.8)^{2t+3} \ ,$$

where t $(t \geq 0)$ is the number of months elapsed since the shop opens.

(i) Find the total profit of the shop in the first 12 months since the shop opens.

(ii) Estimate the rate of change of the total profit of the shop after a very long time.

(9 marks)

a) $\frac{dR}{dt} = \frac{2e^{0.5t}-5e^{-0.5t}}{2e^{0.5t}+5e^{-0.5t}-5}+2$

$\frac{d^2R}{dt^2} = \frac{(e^{0.5t}+2.5e^{-0.5t})(2e^{0.5t}+5e^{-0.5t}-5)-(2e^{0.5t}-5e^{-0.5t})(e^{0.5t}-2.5e^{-0.5t})}{(2e^{0.5t}+5e^{-0.5t}-5)^2}$

$= \frac{2e^{t}+5-5e^{0.5t}+5+12.5e^{-t}-12.5e^{-0.5t}-2e^{t}+5+5-12.5e^{-t}}{(2e^{0.5t}+5e^{-0.5t}-5)^2}$

$= \frac{-5e^{0.5t}-12.5^{-0.5t}+20}{(2e^{0.5t}+5e^{-0.5t}-5)^2}$

For max/min value of $\frac{dR}{dt}$, $\frac{d^2R}{dt^2}=0$

$-5e^{0.5t}-12.5e^{-0.5t}+20=0$

$-5e^{t}+20e^{0.5t}-12.5=0$

$e^{0.5t}=0.7753$, $t=-0.5091$ (rej) or $e^{0.5t}=3.2247$, $t=2.3417$

	$0 \leq t < 2.3417$	$t = 2.3417$	$t > 2.3417$
$\frac{d^2R}{dt^2}$	+	0	−
$\frac{dR}{dt}$	↗	max	↘

$\therefore \frac{dR}{dt}$ is max when $t=2.3417$

$\therefore$ greatest rate of change $= 3.632993162$ thousand dollars per month

$\therefore$ No. <4 thousand dollars per month

Answers written in the margins will not be marked.

b i) $\frac{dP}{dt} = \frac{dR}{dt} - 10(0.8)^{2t+3}$

$$P = \int_0^{12}\left[\frac{2e^{0.5t}-5e^{-0.5t}}{2e^{0.5t}+5e^{-0.5t}-5} + 2 - 10(0.8)^{2t+3}\right]dt$$

$$= \int_0^{12}\frac{2e^{0.5t}-5e^{-0.5t}}{2e^{0.5t}+5e^{-0.5t}-5}\,dt + \int_0^{12}2\,dt - 10\int_0^{12}0.8^{2t+3}\,dt$$

$$= \int_0^{12}\frac{2e^{0.5t}-5e^{-0.5t}}{2e^{0.5t}+5e^{-0.5t}-5}\cdot\frac{1}{0.5(2e^{0.5t}-5e^{-0.5t})}\,d(2e^{0.5t}+5e^{-0.5t}-5)$$

$$+ \Big[2t\Big]_0^{12} - 10\left[\frac{0.8^{2t+3}\ln 0.8}{2}\right]_0^{12}$$

$$= 2\int_0^{12}\frac{1}{2e^{0.5t}+5e^{-0.5t}-5}\,d(2e^{0.5t}+5e^{-0.5t}-5) + 24 - 0.056854985$$

$$= 2\Big[\ln|2e^{0.5t}+5e^{-0.5t}+5|\Big]_0^{12} + 24 - 0.056854985$$

$= 32.3720$ thousand dollars

$\therefore$ total profit $= 32.3720$ thousand dollars

b ii) rate of change of total profit after a very long time

$$= \lim_{t\to\infty}\frac{2e^{0.5t}-5e^{-0.5t}}{2e^{0.5t}+5e^{-0.5t}-5} + 2 - 10(0.8)^{2t+3}$$

$$= \lim_{t\to\infty}\frac{2-5e^{-t}}{2+5e^{-t}-5e^{-0.5t}} + 2 - 10(0.8)^{2t+3}$$

$= 1 + 2$

$= 3$ thousand dollars per month

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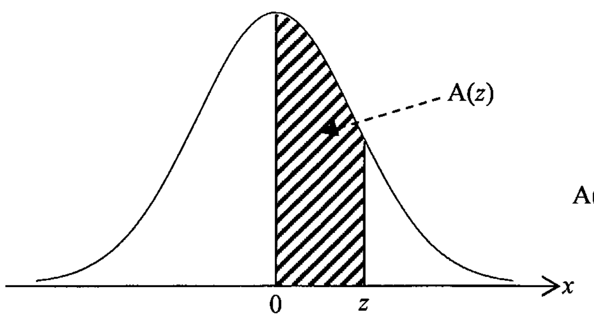
END OF PAPER

Answers written in the margins will not be marked.

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} \, dx$$

Level 5
Exemplar 2

2024-DSE
MATH EP
M1

Please stick the barcode label here.

HONG KONG EXAMINATIONS AND ASSESSMENT AUTHORITY

HONG KONG DIPLOMA OF SECONDARY EDUCATION EXAMINATION 2024

Candidate Number									

MATHEMATICS Extended Part
Module 1 (Calculus and Statistics)
Question-Answer Book

8:30 am – 11:00 am (2½ hours)
This paper must be answered in English

INSTRUCTIONS

(1) After the announcement of the start of the examination, you should first write your Candidate Number in the space provided on Page 1 and stick barcode labels in the spaces provided on Pages 1, 3, 5, 7, 9 and 11.

(2) This paper consists of TWO sections, A and B.

(3) Attempt ALL questions in this paper. Write your answers in the spaces provided in this Question-Answer Book. Do not write in the margins. Answers written in the margins will not be marked.

(4) Graph paper and supplementary answer sheets will be supplied on request. Write your Candidate Number, mark the question number box and stick a barcode label on each sheet, and fasten them with string INSIDE this book.

(5) Unless otherwise specified, all working must be clearly shown.

(6) Unless otherwise specified, numerical answers should be either exact or given to 4 decimal places.

(7) No extra time will be given to candidates for sticking on the barcode labels or filling in the question number boxes after the 'Time is up' announcement.

A031E001

SECTION A (50 marks)

1. The table below shows the probability distribution of a discrete random variable X, where a and b are constants such that $6 < b < 15$.

x	0	3	6	b	15
$P(X = x)$	0.3	a	0.1	0.2	0.2

It is given that $\operatorname{Var}(5X) = 739$.

(a) Find a and b.

(b) Let C be the event that $0 < X \leq 7$.

(i) Let D be the event that $4 < X \leq 15$. Are C and D independent? Explain your answer.

(ii) Let E be an event such that $P(E) \neq 0$. If C and E are mutually exclusive, write down the greatest possible value of $P(E)$.

(7 marks)

(a) $0.3 + a + 0.1 + 0.2 + 0.2 = 1$

$a = 0.2$

$E(X) = 0(0.3) + 3(0.2) + 6(0.1) + b(0.2) + 15(0.2)$

$= 0.2b + 4.2$

$E(X^2) = 0^2(0.3) + 3^2(0.2) + 6^2(0.1) + b^2(0.2) + 15^2(0.2)$

$= 0.2b^2 + 50.4$

$Var(5X) = 25 Var(X) = 739$

$25[E(X^2) - (E(X))^2] = 739$

$25(0.2b^2 + 50.4 - (0.2b + 4.2)^2) = 739$

$0.16b^2 - 1.68b + 32.76 = 29.56$

$0.16b^2 - 1.68b + 3.2 = 0$

$(b-8)(2b-5) = 0$

$b = 8$ or $b = \frac{5}{2}$ (rej)

(b)(i) $P(C) = 0.3 + 0.2 + 0.1 = 0.6$

$P(D) = 0.1 + 0.2 + 0.2 = 0.5$

$P(C \cap D) = 0.1$

$P(C) \times P(D) = (0.6)(0.5) = 0.3$

$\neq P(C \cap D)$ $\therefore$ No

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(ii) $\because$ C and E are mutually exclusive

$P(C) + P(E) \leq 1$

$P(E) \leq 0.4$

$\therefore$ greatest possible of $P(E) = 0.4$

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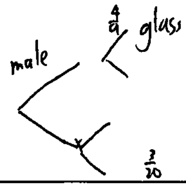

2. In an orchestra, $\frac{3}{5}$ of the members wear glasses. Among the male members, $\frac{4}{9}$ of them wear glasses. The probability that a randomly selected member is a female not wearing glasses is $\frac{3}{20}$.

(a) Given that a randomly selected member does not wear glasses, find the probability that the member is a female.

(b) Find the probability that a randomly selected member is a female wearing glasses.

(6 marks)

(a)

required probability $= \frac{3}{20} \div (1-\frac{3}{5}) = \frac{3}{8}$ //

(b) let x be the probability a selected member is a female

Consider $(1-x)(1-\frac{4}{9}) + \frac{3}{20} = (1-\frac{3}{5})$

$x = \frac{11}{20}$

required probability $= \frac{11}{20} \times (1 - \frac{3}{20} \div \frac{11}{20})$

$= \frac{2}{5}$ //

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3. When a coin is tossed, the probability of getting a tail is p, where $0 < p < 1$. When the coin is tossed 20 times, the ratio of the probability of getting 1 tail to the probability of getting 3 tails is $49 : 57$.

(a) Find p.

(b) The coin is tossed k times. Find the least value of k so that the probability of getting at least 1 tail is greater than 0.85.

(6 marks)

(a) Consider $C_1^{20}(p)(1-p)^{19} : C_3^{20}(p)^3(1-p)^{17} = 49:57$

$\frac{20p(1-p)^{19}}{1140p^3(1-p)^{17}} = \frac{49}{57}$

$\frac{(1-p)^2}{57p^2} = \frac{49}{57}$

$57(1-2p+p^2) = 2793p^2$

$2736p^2+114p-57=0$

$(8p-1)(6p+1)=0$

$p=\frac{1}{8}$ or $p=\frac{-1}{6}$ (rej)

(b) Consider $1-(1-\frac{1}{8})^k > 0.85$

$0.15 > (\frac{7}{8})^k$

$k > \log_{\frac{7}{8}} 0.15$

$k > 14.20729573$

$\therefore$ the least value of k is 15

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4. The weekly revision time of each student in a school follows a normal distribution with a mean of μ hours. A random sample of 81 students is drawn from the school. The mean and the standard deviation of the weekly revision time of these students are 13 hours and 1.75 hours respectively.

(a) The width of a $\beta\%$ confidence interval for μ is 0.7. Find β.

(b) It is given that there are 36 boys in the sample, and the mean and the standard deviation of the weekly revision time of these boys are 12.5 hours and 2 hours respectively. Find the standard deviation of the weekly revision time of the girls in the sample.

[Hint: The sample standard deviation is $\sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)}$.]

(6 marks)

(a) Consider $2 \times z_{\frac{\beta\%}{2}} \times \sqrt{\frac{1.75}{81}} = 0.7$

$\beta\% = 0.4913 \times 2$

$\beta = 98.26$

(b) Considr standard deivatin of boys

$\sqrt{\frac{1}{36-1}\left(\sum_{i=1}^{36} x_i^2 - 36 \times (12.5)^2\right)} = 2$

$\sum_{i=1}^{36} x_i^2 - 5625 = 140$

$\sum_{i=1}^{36} x_i^2 = 5765$

Consider standard deviation of all student

$\sqrt{\frac{1}{81-1}\left(\sum_{i=1}^{81} x_i^2 - 81 \times (13)^2\right)} = 1.75$

$\sum_{i=1}^{81} x_i^2 = 13934$

(sum of revision hours of girls)^2 = $\sum_{i=1}^{45} x_i^2 = 13934 - 5765 = 8169$

standard deviatn of girls

$= \sqrt{\frac{1}{45-1}\left(8169 - 45\left(\frac{81 \times 13 - 36 \times 12.5}{45}\right)^2\right.}$

standard deviation ≈ 1.4206 //

Answers written in the margins will not be marked.

5. Let n be a positive number.

(a) Expand $\frac{2}{e^{nx}}$ in ascending powers of x as far as the term in x^3 .

(b) Consider the expansion of $(1+4x)^m + \frac{2}{e^{nx}}$, where m is a positive integer. The coefficients of x and x^2 in the expansion are 24 and 980 respectively. Find the coefficient of x^3 in the expansion.

(7 marks)

(a) $\frac{2}{e^{nx}} = 2e^{-nx} = 2\left(1 - nx + \frac{(nx)^2}{2} - \frac{(nx)^3}{3!} + \ldots\right)$

$= 2\left(1 - nx + \frac{n^2}{2}x^2 - \frac{n^3}{6}x^3 + \ldots\right)$

$= 2 - 2nx + n^2x^2 - \frac{n^3}{3}x^3 + \ldots$

(b) $(1+4x)^m + \frac{2}{e^{nx}}$

$= \left(1 + C_1^m(4x) + C_2^m(4x)^2 + C_3^m(4x)^3 + \ldots + (4x)^m\right) + 2 - 2nx + n^2x^2 - \frac{n^3}{3}x^3 + \ldots$

$= 3 + (4m - 2n)x + \left(16\frac{(m)(m-1)}{2} + n^2\right)x^2 + \left(64\frac{m(m-1)(m-2)}{3!} - \frac{n^3}{3}\right)x^3$

$\begin{cases} 4m - 2n = 24 & (1) \\ 8m^2 - 8m + n^2 = 980 & (2) \end{cases}$

from (1) $4m - 24 = 2n$

$n = 2m - 12$

put $n = 2m - 12$ int. (2)

$8m^2 - 8m + (2m-12)^2 - 980 = 0$

$12m^2 - 56m - 836 = 0$

$(m-11)(3m+19) = 0$

$m = 11$ or $m = \frac{-19}{3}$ (rej)

$n = 10$

coefficient of $x^3 = 64\frac{(11)(10)(9)}{3!} - \frac{10^3}{3}$

$= \frac{30680}{3}$ //

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6. (a) Let $e^u = (x^2+x+e)^{2x+1}$.

(i) Express u in the form of $\mathrm{p}(x)\ln(\mathrm{q}(x))$, where $\mathrm{p}(x)$ and $\mathrm{q}(x)$ are polynomials.

(ii) Express $\frac{\mathrm{d}}{\mathrm{d}x}e^u$ in terms of x.

(b) The equation of the curve Γ is $y=(x^2+x+e)^{2x+1}$. Denote the point of intersection of Γ and the y-axis by H. Find the equation of the tangent to Γ at H.

(7 marks)

(a)(i) $e^u = (x^2+x+e)^{2x+1}$

$u = (2x+1)\ln(x^2+x+e)$

$p(x) = 2x+1$, $q(x) = x^2+x+e$ //

(ii) $\frac{du}{dx} = (2x+1)\left(\frac{1}{x^2+x+e}\right)(2x+1) + (\ln(x^2+x+e))(2)$

$= \frac{(2x+1)^2}{x^2+x+e} + 2\ln(x^2+x+e)$

$\frac{d}{dx}e^u = \left(\frac{d}{du}e^u\right)\left(\frac{du}{dx}\right)$

$= e^u\left(\frac{(2x+1)^2}{x^2+x+e} + 2\ln(x^2+x+e)\right)$

$= (x^2+x+e)^{2x+1}\left(\frac{(2x+1)^2}{x^2+x+e} + 2\ln(x^2+x+e)\right)$

$= (x^2+x+e)^{2x}(2x+1)^2 + 2(x^2+x+e)^{2x+1}\ln(x^2+x+e)$ //

(b) $H(0, (0^2+0+e)^{2(0)+1})$

$H(0, e)$

$\frac{d}{dx}\Big|_{x=0}(x^2+x+e)^{2x+1} = (0^2+0+e)^{2(0)}(2(0)+1)^2 + 2(0^2+0+e)^{2(0)+1}\ln(0^2+0+e)$

$= (1)(1) + 2(e)(1)$

$= 1+2e$

required equation $= \frac{y-e}{x-0} = 1+2e$

$y-e = (1+2e)x$

$y = (1+2e)x+e$

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7. A computer programme adjusts the length and the breadth of a rectangular digital picture, such that the length of its diagonal remains constant while its breadth decreases at the constant rate of 0.5 cm s^{-1}. Initially, the length and the breadth of the picture are 20 cm and 15 cm respectively. Denote the breadth of the picture by x cm. Find the rate of change of the area of the picture when $x = 7$. (4 marks)

let y cm² be the area

$y = 20(x)$

$\frac{dy}{dx} = 20$

$\frac{dy}{ds} = \frac{dy}{dx}\left(\frac{dx}{ds}\right)$

$\left.\frac{dy}{ds}\right|_{x=7} = \left(\left.\frac{dy}{dx}\right|_{x=7}\right)\left(\left.\frac{dx}{ds}\right|_{x=7}\right) = 20(-0.5)$

$= -10\text{ cm}^2\text{ s}^{-1}$

rate of change of area when $x=7$ is $-10\text{ cm}^2\text{ s}^{-1}$

Answers written in the margins will not be marked.

8. (a) Using the Standard Normal Distribution Table on page 24, evaluate $\int_0^{0.5} e^{\frac{-x^2}{2}} dx$.

(b) Consider the curve $C: y=(2x-1)e^{\frac{-x^2}{2}}$, where $x \geq 0$. Using the result of (a), find the area of the region bounded by C, the x-axis and the y-axis.

(7 marks)

(a) $\int_0^{0.5} \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} = 0.1915$

$\left(\frac{1}{\sqrt{2\pi}}\right)\int_0^{0.5} e^{-\frac{x^2}{2}} = 0.1915$

$\int_0^{0.5} e^{-\frac{x^2}{2}} = 0.1915(\sqrt{2\pi})$

≈ 0.4800

(b) Consider $y=0$

$(2x-1)(e^{-\frac{x^2}{2}})=0$

$x=\frac{1}{2}$

required area $= -\int_0^{0.5}\left((2x-1)e^{-\frac{x^2}{2}}\right)dx$

$= -\int_0^{0.5}\left(2xe^{-\frac{x^2}{2}} - e^{-\frac{x^2}{2}}\right)dx$

let $u = -\frac{x^2}{2}$

$du = -x\ dx$

when $x=0$ $u=1$

when $x=0.5$ $u=-\frac{1}{8}$

$= \int_1^{-\frac{1}{8}} -2e^u\ du + \int_0^{0.5} e^{-\frac{x^2}{2}} dx$

$\approx 2[e^u]_0^{-\frac{1}{8}} + 0.1915(\sqrt{2\pi})$

$\approx 0.24501311 9$

≈ 0.2450 //

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SECTION B (50 marks)

9. The weight of each pumpkin in a large market follows a normal distribution with a mean of μ kg and a standard deviation of σ kg. It is given that 30.85% of the pumpkins in the market each weighs more than 5.7 kg while 78.88% of the pumpkins each weighs between $(\mu - 1.5)$ kg and $(\mu + 1.5)$ kg .

   (a) Find μ and σ. (3 marks)

   (b) Suppose that 16 pumpkins are randomly chosen in the market. Find the probability that the mean weight of these pumpkins does not exceed 5.4 kg. (2 marks)

   (c) The following table shows the grades and the prices of the pumpkins in the market.

Weight of a pumpkin (W kg)	$W \leq 3.6$	$3.6 < W \leq 5.7$	$W > 5.7$
Grade	C	B	A
Price ($)	50	80	100

   Suppose that 8 pumpkins are randomly chosen in the market and these pumpkins are put into a trolley.

   (i) Find the expected price of the pumpkins in the trolley.

   (ii) Find the probability that there are at least 5 grade B pumpkins and at least 1 grade A pumpkin in the trolley.

   (6 marks)

(a) $\begin{cases} P\left(Z > \frac{5.7-\mu}{\sigma}\right) = 30.85\% \\ P\left(\frac{\mu-1.5-\mu}{\sigma} < Z < \frac{\mu+1.5-\mu}{\sigma}\right) = 78.88\% \end{cases}$

$\begin{cases} \frac{5.7-\mu}{\sigma} = 0.5 \quad ① \\ \frac{1.5}{\sigma} = 1.25 \quad ② \end{cases}$

From ② $\sigma = 1.2$

put $\sigma = 1.2$ into ① $\mu = 5.1$

(b) required probability $= P(\bar{X} \leq 5.4)$

$= P\left(Z \leq \frac{5.4-5.1}{\frac{1.2}{\sqrt{16}}}\right)$

$= P(Z \leq 1)$

$= 0.5 + 0.3413$

$= 0.8413$

Answers written in the margins will not be marked.

5 1

(c)(i) $P(W > 5.7) = 30.85\%$

$P(W \leq 3.6) = P(Z < \frac{3.6-5.1}{1.2}) = 0.1056$

$P(3.6 < W \leq 5.7) = 1 - 0.1056 - 0.3085 = 0.5859$

expected price $= 8\left((0.1056)(50) + (0.5859)(80) + (0.3085)(100)\right)$

$= \$664.016$

(ii) required probability

$= C^8_1(0.3085)C^7_5(0.5859)^5(0.1056)^2 + C^8_2(0.3085)^2C^6_5(0.5859)^5(0.1056) + C^8_3(0.3085)^3(0.5859)^5$

$+ C^8_1(0.3085)C^7_6(0.5859)^6(0.1056) + C^8_1(0.3085)(0.5859)^7 + C^8_2(0.3085)^2(0.5859)^6$

≈ 0.3311

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10. A courier delivers goods every day. The number of delays in delivery on a day follows a Poisson distribution with a mean of 1.6 . A day is regarded as *smooth* if there are fewer than 3 delays on that day.

(a) Find the probability that a certain day is *smooth.* (2 marks)

(b) Find the probability that all the 7 days in a certain week are *smooth.* (2 marks)

(c) Given that all the 7 days in a certain week are *smooth*, find the probability that there are exactly 10 delays in that week. (4 marks)

(d) Given that there are no delays in at least 2 days in a certain week, find the probability that all the 7 days in that week are *smooth.* (4 marks)

(a) required probability $= e^{-1.6}\left(1+1.6+\frac{(1.6)^2}{2!}\right)$

≈ 0.783358489

≈ 0.7834

(b) required probability $= (0.783358489)^7$

≈ 0.181018883

≈ 0.1810

(c) required probability

$$= \frac{C_3^7\left(\frac{e^{-1.6}(1.6)^2}{2!}\right)^3\left(\frac{e^{-1.6}(1.6)}{1!}\right)^4 + C_1^7\left(\frac{e^{-1.6}(1.6)^0}{0!}\right)C_2^6\left(\frac{e^{-1.6}(1.6)}{1!}\right)^2\left(\frac{e^{-1.6}(1.6)^2}{2!}\right)^4 + C_2^7\left(\frac{e^{-1.6}(1.6)^0}{0!}\right)^2\left(\frac{e^{-1.6}(1.6)^2}{2!}\right)^5}{0.181018883}$$

≈ 0.096294543

≈ 0.0963

(d) required probability $= \dfrac{(0.783358489)^7}{1-(1-0.783358489)^7-C_1^7(0.783358489)(1-0.783358489)^6}$

≈ 0.18112562

≈ 0.1811

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11. The accumulative rainfall of city M on a certain day increases at a rate of P mm per hour. It is given that

$$P = a(-t^2 + 10t + 8)e^{bt} ,$$

where a and b are constants, and t $(0 \le t \le 4)$ is the number of hours elapsed since 7 am on that day. It is found that $\ln\left(\dfrac{P}{-t^2+10t+8}\right)$ is a linear function of t, and the graph of this linear function passes through the point $(3, -0.1)$ and the intercept on the horizontal axis is 2.5 . $(2.5, 0)$

(a) Express $\ln\left(\dfrac{P}{-t^2+10t+8}\right)$ as a linear function of t. (1 mark)

(b) Find the exact values of a and b. (3 marks)

(c) Using the trapezoidal rule with 4 sub-intervals, estimate the accumulative rainfall of city M from 7 am to 11 am on that day. (2 marks)

(d) The accumulative rainfall of city N on the same day increases at a rate of Q mm per hour. It is given that

$$Q = \frac{16(2t+5)e^{0.4t}}{4te^{0.4t}+3} ,$$

where t $(0 \le t \le 4)$ is the number of hours elapsed since 7 am on that day.

(i) Find $\int Q \, dt$.

(ii) Someone claims that the sum of the accumulative rainfalls of city M and city N from 7 am to 11 am on that day is greater than 160 mm . Do you agree? Explain your answer. (8 marks)

(a) $P = a(-t^2+10t+8)e^{bt}$

$\dfrac{P}{-t^2+10t+8} = ae^{bt}$

$\ln\left(\dfrac{P}{-t^2+10t+8}\right) = bt + \ln a$

(b) $\begin{cases} b(3) + \ln a = -0.1 \\ b(2.5) + \ln a = 0 \end{cases}$

By solving

$b = -0.2 \quad \ln a = 0.5$

$a = e^{\frac{1}{2}}$

(c) let $(e^{\frac{1}{2}})(-t^2+10t+8)e^{-0.2t}$ be $f(x)$

estimate value $= \dfrac{\left(\frac{4-0}{4}\right)}{2}\left(f(0)+f(4)+2\left(f(1)+f(2)+f(3)\right)\right)$

≈ 94.1599625

≈ 94.1600 mm

Answers written in the margins will not be marked.

(d)(i) let $u = 4te^{0.4t} + 3$

$\frac{du}{dt} = 4t(0.4)(e^{0.4t}) + (e^{0.4t})(4)$

$= 4(e^{0.4t})(0.4t+1)$

$\int Q\,dt$

$= \int \frac{20}{u}\,dt$

$= 20 \ln |4te^{0.4t}+3| + C$ //

(ii) $P = (e^{\frac{1}{2}})(-t^2+10t+8)(e^{-0.2t})$

$\frac{dP}{dt} = (e^{\frac{1}{2}})[(-0.2)(e^{-0.2t})(-t^2+10t+8) + (e^{-0.2t})(-2t+10)]$

$= (e^{\frac{1}{2}})(e^{-0.2t})(0.2t^2-3t+8.4)$

$\frac{d^2P}{dt^2} = (e^{\frac{1}{2}})[(e^{-0.2t})(0.4t-3) + (0.2t^2-3t+8.4)(-0.2)(e^{-0.2t})]$

$= (e^{\frac{1}{2}})(e^{-0.2t})(-0.04t^2+t-4.68)$

Consider $-0.04t^2+t-4.68=0$

$t = \frac{-(1) \pm \sqrt{(1)^2-4(0.04)(4.68)}}{2(-0.04)}$

$t = 6.235017957$ or $t = 18.76498204$

$-0.04t^2+t-4.68 < 0$ for $0 \le t \le 4$

$\therefore$ P is under-estimate.

sum of accumulative rainfalls $\approx 94.159925 + [20 \ln |4te^{0.4t}+3|]_6^4$

$\approx 94.159925 + 66.22266184$

≈ 160.3825868

≈ 160.3827

> 160 mm and P is under-estimate

$\therefore$ Yes

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12. Let R be the total revenue (in thousand dollars) of an online shop. It is given that

$$\frac{dR}{dt} = \frac{2e^{0.5t} - 5e^{-0.5t}}{2e^{0.5t} + 5e^{-0.5t} - 5} + 2 ,$$

where t $(t \geq 0)$ is the number of months elapsed since the shop opens.

(a) Does the greatest rate of change of the total revenue of the shop exceed 4 thousand dollars per month? Explain your answer. (4 marks)

(b) Let P be the total profit (in thousand dollars) of the shop. It is given that

$$\frac{dP}{dt} = \frac{dR}{dt} - 10(0.8)^{2t+3} ,$$

where t $(t \geq 0)$ is the number of months elapsed since the shop opens.

(i) Find the total profit of the shop in the first 12 months since the shop opens.

(ii) Estimate the rate of change of the total profit of the shop after a very long time.

(9 marks)

(a) $\frac{d^2R}{dt^2} = \frac{(2e^{0.5t}+5e^{-0.5t}-5)(e^{0.5t}+2.5e^{-0.5t}) - (2e^{0.5t}-5e^{-0.5t})(e^{0.5t}-2.5e^{-0.5t})}{(2e^{0.5t}+5e^{-0.5t}-5)^2}$

$= \frac{2e^{t}+5-5e^{0.5t}+5+12.5e^{-t}-12.5e^{-0.5t}-(2e^{t}-5-5+12.5e^{-t})}{(2e^{0.5t}+5e^{-0.5t}-2)^2}$

$= \frac{20-5e^{0.5t}-12.5e^{-0.5t}}{(2e^{0.2t}+5e^{-0.5t}-2)^2}$

Consid. $\frac{d^2R}{dt^2}=0$ $\quad 20-5e^{0.5t}-12.5e^{-0.5t}=0$

$e^{0.5t}=0.775255128$ (rej) or $e^{0.5t}=3.224744871$

$t=2.341709273$

t	$0 \leq t < 2.3417$	$t = 2.3417$

Answers written in the margins will not be marked.

(b)(i) total profit $= \int_0^{12} \frac{dP}{dt}$

let $u = 2e^{0.5t} + 5e^{-0.5t} - 5$

$du = (0.5)(2e^{0.5t} - 5e^{-0.5t})\,dt$

when $t=12$ $u = 2e^6 + 5e^{-6} - 5$

when $t=0$ $u = 2$

$= \int_2^{2e^6+5e^{-6}-5} 2\,\frac{1}{u}\,du + \int_0^{12} 2\,dt - \int_0^{12} 10(0.8)^{2t+3}\,dt$

$= 2[\ln|u|]_2^{2e^6+5e^{-6}-5} + [2t]_0^{12} - 10\left(\frac{1}{2}\right)\left[\frac{0.8^{2t+3}}{\ln 8}\right]_0^{12}$

≈ 38.38627662

≈ 38.3863 thousand dollar

(ii) $\frac{d^2P}{dt^2} = \frac{d^2R}{dt^2} - 10(0.8)^{2t+3}(\ln 0.8)(2)$

$= \frac{d^2R}{dt^2} - (20\ln 0.8)(0.8^{2t+3})$

$\lim_{t\to\infty} \frac{d^2R}{dt^2} - (20\ln 0.8)(0.8^{2t+3})$

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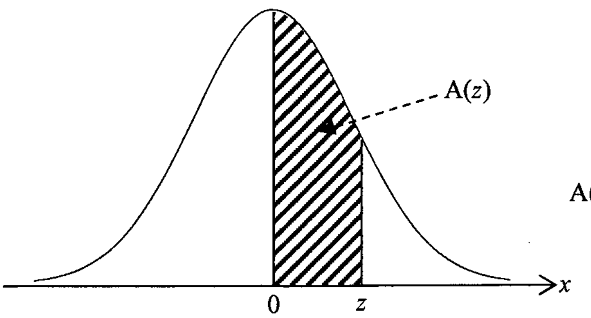
END OF PAPER

Answers written in the margins will not be marked.

Standard Normal Distribution Table

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998

Note : An entry in the table is the area under the standard normal curve between $x = 0$ and $x = z$ $(z \geq 0)$. Areas for negative values of z can be obtained by symmetry.



$$A(z) = \int_0^z \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}} dx$$