PURE MATHEMATICS

ADVANCED LEVEL

OBJECTIVE

The objective of the examination is to test the understanding of basic mathematical concepts and their applications.

THE EXAMINATION

The examination will consist of two equally-weighted papers of 3 hours each. In each paper there will be two sections. Section A (40%) will consist of 6-8 short questions, all of which are to be attempted. Section B (60%) will consist of 5 long questions, of which candidates will be required to answer 4.

Notes:
1. The following syllabus is an examination syllabus and is not to be interpreted as an exact and exhaustive teaching syllabus.
2. The syllabus is to be viewed as a whole and, in particular, the length of a section is not an indication of its weight in the examination.
3. Candidates are required to be familiar with the use of the language of sets. In writing solutions, candidates are expected to be able to reason logically, distinguishing assumptions from conclusions. They should understand the nature of proof (direct or by contradiction) and disproof and the meaning of the terms ‘sufficient/necessary condition’ and ‘converse/contrapositive statement’ of a conditional statement.
4. Unless the terms of the questions impose specific limitations,
    a. a candidate may use any appropriate method,
    b. calculators* may be used in the examination.

THE SYLLABUS

	Syllabus	*Notes*
1.	Mathematical induction.	Understanding and use of the principle and its variations (excluding backward induction).
2.	Inequalities.	Including A.M. $\geq$ G.M. and Cauchy-Schwarz's inequality.
3.	The binomial theorem for positive integral indices.	Simple properties of the binomial coefficients.
4.	Complex numbers.	Arithmetic of complex numbers. Modulus, argument and conjugate. Argand diagrams. Triangle inequality. Simple applications in plane geometry.
	De Moivre's theorem for rational indices.	Including applications to trigonometric identities and *n*th roots of a complex number.
5.	Polynomials with real coefficients in one variable.	Division algorithm and remainder theorem. Euclidean algorithm. Knowledge of the following expected: (i) $\deg(\mathrm{fg}) = \deg \mathrm{f} + \deg \mathrm{g}$, (ii) $\deg(\mathrm{f} + \mathrm{g}) \leq \max(\deg \mathrm{f}, \deg \mathrm{g})$.
	Rational functions.	Including representation by partial fractions.
	Polynomial equations with real coefficients in one variable.	Relations between coefficients and roots. Complex roots occurring in conjugate pairs.

	Syllabus	*Notes*
6.	Matrices.	Addition and multiplication. Scalar multiplication. Transpose.
	Square matrices of orders 2 and 3.	Singular and non-singular matrices. Inverses, determinants and knowledge of their simple properties, including $\det(AB) = (\det A)(\det B)$.
	Applications to 2-dimensional geometry.	Matrix representation of reflections, rotations, enlargements, shears, translations and their compositions.
7.	System of linear equations in two or three unknowns.	Gaussian elimination and Echelon form. Existence and uniqueness of solution.
8.	Conic sections in rectangular coordinates.	Emphasis on equations of conics in standard position. Parametric representation. Tangents and normals. Asymptotes of a hyperbola. Knowledge of eccentricity, focus and directrix not emphasized.
	Plane curves in rectangular coordinates.	Tangents and normals. Simple problems on loci. Use of parameters.

	Syllabus	*Notes*
9.	Functions and their graphs.	Definition of a function. Injective, surjective and bijective functions. Composition of functions. Inverse functions. Odd, even and periodic functions.
	Elementary functions.	Algebraic functions. Trigonometric functions and their inverses (including compound angle formulas and related formulas). Exponential and logarithmic functions.
10.	Intuitive concept of limit, and based on that, continuity and differentiability.	Concept of sequences and series. Limit of a function and of a sequence. Arithmetic operations on limits. Knowledge that a bounded monotonic sequence converges and familiarity with the use of the sandwich theorem expected. Convergence tests for series not required.
11.	Differentiation.	Differentiation of elementary functions, of sums, products and quotients of functions, of composite, inverse and implicit functions. Knowledge of the Mean Value Theorem. Higher order derivatives. Knowledge of Leibniz's Theorem.
	Applications of differentiation.	Maxima and minima, curve sketching in the rectangular coordinate system (including point of inflexion) and rates of change. Use of L'Hospital's rule.

	Syllabus	*Notes*
12.	Integration.	The notion of integral as the limit of a sum. Simple properties of integrals. Knowledge of the fundamental theorem of integral calculus: $\frac{\mathrm{d}}{\mathrm{d}x}\int_a^x \mathrm{f}(t)\,\mathrm{d}t = \mathrm{f}(x)$ and its application to evaluation of integrals. Indefinite integrals.
	Methods of integration.	Integration by substitution, by partial fractions and by parts. Reduction formulas.
	Applications of integration.	Plane areas and volumes, including use of parameters.

* See Regulation 5.15.