APPLIED MATHEMATICS

ADVANCED LEVEL

INTRODUCTION

This syllabus serves to examine candidates' knowledge and skills in introductory mathematical and statistical methods, and their applications. For applications of mathematical methods, candidates are required to have specific knowledge in theoretical mechanics. In addition, candidates should understand the methods well enough to be able to apply them in other disciplines, but in such cases no prerequisite knowledge of these other disciplines is required. Candidates are expected to have acquired knowledge of the subject matter in the Mathematics syllabus of the Hong Kong Certificate of Education Examination. In addition, they are also expected to have a knowledge of the following topics : exponential and logarithmic functions, trigonometry (including compound angle formulas, sum and product formulas), basic operation of complex numbers in standard and polar forms, elementary calculus.

THE EXAMINATION

The examination will consist of two equally-weighted papers of 3 hours each. In each paper there will be two sections. Section A (40%) will consist of 6-8 short questions, all of which are to be attempted. Section B (60%) will consist of 5 long questions, of which candidates will be required to answer 4.

Note : Unless the terms of the questions impose specific limitations,

a. a candidate may use any appropriate method,
b. calculators* may be used in the examination.

THE SYLLABUS

Notes :

1. The following syllabus is an examination syllabus and is not to be interpreted as an exact and exhaustive teaching syllabus.
2. The syllabus is to be viewed as a whole and, in particular, the length of a section is not an indication of its weight in the examination.

* See Regulation 5.15.

Topics	*Explanatory notes*
Paper 1	
I. Theoretical Mechanics	
1. **Vectors in $\mathbf{R}^2$ and $\mathbf{R}^3$**	
Vector addition and subtraction. Multiplication by a scalar. Resolution of vectors.	
Position vectors and unit vectors.	Use of the unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$. Vector equation of a line.
Scalar product and orthogonality. Vector product and parallelism.	$\mathbf{a} \times \mathbf{b} = \lvert \mathbf{a} \rvert \lvert \mathbf{b} \rvert \sin\theta \, \mathbf{n}$. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$. $\mathbf{a} \times (\mathbf{b}+\mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$. (Proof not required)
Differentiation of a vector function with respect to a scalar variable.	Knowledge of the following formulas is expected : $\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{u}+\mathbf{v}) = \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} + \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$ $\frac{\mathrm{d}}{\mathrm{d}t}(\lambda\mathbf{u}) = \frac{\mathrm{d}\lambda}{\mathrm{d}t}\mathbf{u} + \lambda\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}$

	Topics	*Explanatory notes*
		$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{u}\cdot\mathbf{v}) = \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}\cdot\mathbf{v} + \mathbf{u}\cdot\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$ $\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{u}\times\mathbf{v}) = \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}\times\mathbf{v} + \mathbf{u}\times\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$
	Integration of a vector function with respect to a scalar variable.	Knowledge of line integrals is not required.
2.	**Statics**	
	Force. Moment and couple. Resultant of system of forces.	Reduction of a system of forces to a resultant and a couple.
	Equilibrium of particles and rigid bodies under a system of coplanar forces.	Candidates are expected to be able to set up independent equations and inequalities from a given physical situation.
3.	**Kinematics**	
	Motion of a particle in a plane.	Displacement, velocity and acceleration. Angular displacement, angular velocity and angular acceleration. Velocity and acceleration components in tangential-normal form are not required.
	Relative motion.	Simple problems on the motion of two bodies.

	Topics	*Explanatory notes*
	Resolution of velocity and acceleration along and perpendicular to radius vector.	Derivation and application of the formulas $\dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\theta}\,\mathbf{e}_\theta$ $\ddot{\mathbf{r}} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}t}(r^2\dot{\theta})\mathbf{e}_\theta$. Detailed knowledge of orbit problems is not required.
4.	**Newton's laws of motion**	Questions involving variable mass will not be set.
	The three laws of motion. Work. Energy, momentum and their conservation laws.	
	Rectilinear motion of a particle.	
	Simple harmonic motion. Damped oscillations.	
	Motion of a particle in a plane.	Questions involving Cartesian or polar coordinates may be set.
	Motion of projectile under gravity.	Two-dimensional problems involving resistive media will not be set.
	Circular motion. Motion in a vertical circle.	

Topics	*Explanatory notes*
5. **Impact**	
Direct and oblique impacts.	Questions involving impulsive tensions will not be set.
Elastic and inelastic impacts.	Coefficient of restitution.
6. **Friction**	
Laws of static and kinetic friction. Coefficient of friction. Angle of friction.	Application to statical and dynamical problems involving particles and rigid bodies.
Limiting positions of equilibrium.	Knowledge of stability of equilibrium is not expected.
7. **Motion of a rigid body**	
Rigid body as a system of particles. Centre of mass.	Centre of mass of uniform bodies using symmetry or integration. Centre of mass of a composite body.
Moment of inertia. Parallel and perpendicular axes theorems.	Moment of inertia of uniform bodies using integration. Moment of inertia of a composite body.

Topics	*Explanatory notes*
Angular momentum.	Law of angular momentum. Conservation law.
Potential and kinetic energy.	
Motion of a rigid body about a fixed axis.	

Paper 2

II. Differential Equations

For all types of differential equations below, candidates are expected to be familiar with examples of their formulation from practical situations, as well as the interpretation of their solutions. However, candidates are not expected to have particular knowledge of a practical situation, but will be given a description sufficient to formulate an equation.

1. **First order differential equations**

Solution of	
(a) equations with variables separable,	
(b) linear equations $\frac{\mathrm{d}y}{\mathrm{d}x}+p(x)\,y=q(x)$.	Use of the integrating factor is expected. Candidates will be expected to be able to use a given substitution that reduces an equation to one of the above types.

Topics	*Explanatory notes*
2. **Second order differential equations**	
Solution of	
(a) homogeneous equations with constant coefficients $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=0$,	Use of the auxiliary equation is expected.
(b) non-homogeneous equations with constant coefficients $a\frac{d^2y}{dx^2}+b\frac{dy}{dx}+cy=f(x)$,	$f(x)$ will only be of the form x^n, $\cos px$, $\sin px$, e^{px}, or a linear combination of these functions. Candidates will be expected to be able to deal with cases when p is a root of the auxiliary equation. The method of undetermined coefficients is expected. Candidates will be expected to be able to use a given substitution that reduces an equation to one of the above types. Candidates will be expected, for both homogeneous and non-homogeneous equations, to be able to find the general solution, and the solutions to either an initial value problem or a boundary value problem.

	Topics	*Explanatory notes*
	(c) system of two first order differential equations.	Only simple systems which may be reduced by elimination to a second order linear differential equation are expected.
III.	Numerical Methods	
	1. **Approximation**	
	Interpolating polynomials.	Including Lagrangian interpolating polynomial and its error term.
	Approximation of functions using Taylor's expansion.	Candidates will be expected to estimate the error.
	2. **Numerical integration**	
	Trapezoidal rule, Simpson's rule and their composite formulas.	Candidates will be expected to estimate the error. (Derivation of the error formulas is not required.) Candidates may be expected to employ other integration rules but not to derive them. Such rules will be given in the examination.
	3. **Numerical solution of equations**	The determination of the order of convergence is not required.
	Method of false position and secant method.	Detailed treatment of the convergence and error estimation is not required.

Topics	*Explanatory notes*
Method of fixed-point iteration. Newton's method.	Candidates will be expected to estimate the error. The following sufficient condition of convergence for the method of fixed-point iteration is expected : (1) The iterative function $g(x)$ is continuous and differentiable on an interval I ; (2) $g(x) \in I$ for all x in I ; (3) There exists a positive constant k such that $\lvert g'(x) \rvert \leq k < 1$ for all x in I . Treatment of Newton's method as a particular example of the method of fixed-point iteration.

IV. Probability and Statistics

1. **Basic statistical measures**

Mean, mode and median. Standard deviation and variance.	Candidates are expected to know the calculation of the combined mean and variance from two or more sets of data.

2. **Probability laws**

Sample points, sample space and events. Equally likely events. Ways of counting.	Permutations and combinations.

	Topics	*Explanatory notes*
	Sum and product laws. Mutually exclusive events and independent events. Conditional probability.	Use of tree diagrams. Candidates are expected to know the formulas (i) $P(A') = 1 - P(A)$ (ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (iii) $P(A \cap B) = P(A)\,P(B \mid A)$
	Bayes' Theorem.	Derivation of the theorem is not required.
3.	**Probability distributions**	
	Random variables.	Probability and distribution functions of a discrete random variable. Probability density and distribution functions of a continuous random variable. Expectation and Variance. Candidates are expected to know the formulas $E(aX + bY) = aE(X) + bE(Y)$ and $Var(aX + bY) = a^2Var(X) + b^2Var(Y)$ for independent random variables X and Y.
	Binomial and Normal distributions.	Bernoulli trials. Binomial probability. Use of standard normal distribution table. The normal distribution as an approximation to the binomial distribution, including the use of the continuity correction.

Topics	*Explanatory notes*
	Derivation of mean and variance in normal distribution is not required. Knowledge that a linear combination of two independent normally distributed variables is also normally distributed is assumed. Particular knowledge of other probability distributions is not required.
4. **Statistical inference**	
Estimate of a population mean from a random sample.	
Confidence interval for the mean of a normal population with known variance.	
Hypothesis testing.	The notions of test statistic, critical region and significance level. Type I and Type II errors. Only one-sample problems involving the use of normal distribution will be set.