New Senior Secondary Mathematics Curriculum

Public Examination of Extended Part

HKEAA
November 2008
Public Examination of Extended Part

- Result of Questionnaires after Third Consultation
- Examination Format
- Sample Paper
- Draft Level Descriptor
  { M1 & M2 }
- Annotated Examples
Results of Questionnaires after Third Consultation
The proposed assessment objectives of the public assessment are in alignment with the aims and objectives of the curriculum. (M1)
The proposed paper structure for the public examination is appropriate. (M1)
The proposed duration of the public examination is appropriate. (M1)
The proposed assessment objectives of the public assessment are in alignment with the aims and objectives of the curriculum. (M2)
The proposed paper structure for the public examination is appropriate. (M2)
The proposed duration of the public examination is appropriate. (M2)
**Examination Format**

### Module 1 (Calculus and Statistics)

<table>
<thead>
<tr>
<th>Component</th>
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<tr>
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<td>100%</td>
<td>2 1/2 hours</td>
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### Module 2 (Algebra and Calculus)

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Some Features in Both M1 and M2

- Compulsory Part + Secondary 1-3 Mathematics Curriculum
- Two Sections
  - Section A: 8-12 short questions
  - Section B: 3-5 long questions
- **All** the questions to be attempted
- Each question may not carry the same mark
Some of the contrasts between the content of the HKDSE Mathematics (M1) and the present HKALE Mathematics and Statistics

- Expectation
- Sampling Distribution
- Point Estimates
- Confidence Interval
Some of the contrasts between the content of the HKDSE Mathematics (M2) and the present HKALE Pure Mathematics

- Vectors in $\mathbb{R}^3$
- Vector Product
Section A

1. Expand the following in ascending powers of $x$ as far as the term in $x^2$:

(a) $e^{-2x}$;

(b) $\frac{(1+2x)^6}{e^{2x}}$. 
3. A political party studied the public view on a certain government policy. A random sample of 150 people was taken and 57 of them supported this policy.

(a) Estimate the population proportion supporting this policy.

(b) Find an approximate 90% confidence interval for the population proportion.
5. A manufacturer produces a large batch of light bulbs, with a mean lifetime of 640 hours and a standard deviation of 40 hours. A random sample of 25 bulbs is taken. Find the probability that the sample mean lifetime of the 25 bulbs is greater than 630 hours.
7. The random variable $X$ has probability distribution $P(X = x)$ for $x = 1, 2$ and $3$ as shown in the following table.

<table>
<thead>
<tr>
<th>$x$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td>0.1</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Calculate
(a) $E(X)$,

(b) $\text{Var}(3 - 2X)$.
Section B
12.

(a) Let \( f(t) \) be a function defined for all \( t \geq 0 \). It is given that

\[
f'(t) = e^{2bt} + ae^{bt} + 8
\]

where \( a \) and \( b \) are negative constants and \( f(0) = 0 \), \( f'(0) = 3 \) and \( f'(1) = 4.73 \).

(i) Find the values of \( a \) and \( b \).

(ii) By taking \( b = -0.5 \), find \( f(12) \).
(b) Let \( g(t) \) be another function defined for all \( t \geq 0 \). It is given that
\[
g'(t) = \frac{33}{10} te^{-kt},
\]
where \( k \) is a positive constant. Figure 1 shows a sketch of the graph of \( g'(t) \) against \( t \). It is given that \( g'(t) \) attains the greatest value at \( t = 7.5 \) and \( g(0) = 0 \).
(i) Find the value of \( k \).

(ii) Use the trapezoidal rule with four sub-intervals to estimate \( g(12) \).

(iii) From the estimated value obtained in (b)(ii) and Figure 1, Jenny claims that \( g(12) > f(12) \). Do you agree? Explain your answer.
14. The Body Mass Index (BMI) value (in \( \text{kg/m}^2 \)) of children aged 12 in a city are assumed to follow a normal distribution with mean \( \mu \ \text{kg/m}^2 \) and standard deviation 4.5 \( \text{kg/m}^2 \).

(a) A random sample of nine children aged 12 is drawn and their BMI values (in \( \text{kg/m}^2 \)) are recorded as follows:
16.0, 18.3, 15.2, 17.8, 19.5, 15.9, 18.6, 22.5, 23.6

(i) Find an unbiased estimate for \( \mu \).

(ii) Construct a 95% confidence interval for \( \mu \).
(b) Assume $\mu = 18.7$. If a random sample of 25 children aged 12 is drawn and their BMI values are recorded, find the probability that the sample mean is less than $17.8 \text{ kg/m}^2$. 
(c) A child aged 12 having a BMI value greater than 25 kg/m² is said to be overweight. Children aged 12 are randomly selected one after another and their BMI values are recorded until two overweight children are found. Assume that $\mu = 18.7$.

(i) Find the probability that a selected child is overweight.

(ii) Find the probability that more than eight children have to be selected in this sampling process.

(iii) Given that more than eight children will be selected in this sampling process, find the probability that exactly ten children are selected.
Draft Level Descriptors (M1)

The typical performance of candidates at this level:
Level 5

- demonstrate comprehensive knowledge and understanding of the calculus and statistics concepts in the curriculum by applying them successfully at a sophisticated level to a wide range of unfamiliar situations
- communicate and express views and arguments precisely and logically using mathematical language, notations, tables, diagrams and graphs
- formulate mathematical models successfully in complex situations, employ appropriate strategies to arrive at a complete solution, and evaluate the significance and reasonableness of results
- integrate knowledge and skills from different areas of the curriculum in handling complex tasks using a variety of strategies
Level 4

- demonstrate sound knowledge and understanding of the calculus and statistics concepts in the curriculum by applying them successfully to unfamiliar situations
- communicate and express views and arguments accurately using mathematical language, notations, tables, diagrams and graphs
- formulate mathematical models successfully in unfamiliar situations, employ appropriate strategies to arrive at a solution, and explain the significance and reasonableness of results
- integrate knowledge and skills from different areas of the curriculum in handling a range of tasks
Level 3

- demonstrate adequate knowledge and understanding of the calculus and statistics concepts in the curriculum by applying them successfully to familiar and some unfamiliar situations
- communicate and express views and arguments appropriately using mathematical language, notations, tables, diagrams and graphs
- formulate mathematical models in familiar situations, employ relevant mathematical techniques to arrive at some results, and are aware of the significance and reasonableness of results
- integrate knowledge and skills from different areas of the curriculum in handling mathematical tasks in explicit situations
- apply calculus and statistics techniques in solving simple real-life problems
Level 2

- demonstrate basic knowledge and understanding of the calculus and statistics concepts in the curriculum by employing simple algorithms, formulas and procedures in performing routine tasks
- communicate and express relevant views using mathematical language, notations, tables, diagrams and graphs
- define appropriate parameters or variables in simple tasks and employ elementary mathematics techniques to arrive at some results
- use differentiation and integration in handling straightforward and clearly defined problems
- apply some properties of binomial, geometric, Poisson and normal distributions in simple cases
Level 1

- demonstrate elementary knowledge and understanding of the calculus and statistics concepts in the curriculum by performing straightforward procedures according to direct instructions
- communicate and express simple ideas using mathematical language, notations, tables, diagrams and graphs
- define some parameters or variables that are relevant to the context
- find derivatives and integrals of simple functions
- find probabilities in straightforward cases
Annotated Examples (M1)
Level 5 (Q13)

There are 80 operators in an emergency hotline centre. Assume that the number of incoming calls for the operators are independent and the number of incoming calls for each operator is distributed as Poisson with mean 6.2 in a ten-minute time interval (TMTI). An operator is said to be \textit{idle} if the number of incoming calls received is less than three in a certain TMTI.

(a) Find the probability that a certain operator is \textit{idle} in a TMTI.

(b) Find the probability that there are at most two \textit{idle} operators in a TMTI.

(c) A manager, Calvin, checks the numbers of incoming calls of the operators one by one in a TMTI. What is the least number of operators to be checked so that the probability of finding an \textit{idle} operator is greater than 0.9?
Level 5 (Q13)

Formulate mathematical models successfully

Apply Poisson and binomial probabilities accurately

Demonstrate comprehensive knowledge and understanding of the statistics concepts

\[ X \sim P(6.2) \]

(a) \[ P(X \geq 3) \]

\[ = e^{-6.2} \left( 1 + \frac{6.2}{1} + \frac{6.2^2}{2} \right) \]

\[ = 0.053611554 \approx 0.0536 \]

(b) 

\[ = 0.19079584 \approx 0.1908 \]

(c) 

\[ n \ln 0.1 = 0.0451735 \]

\[ n = \frac{0.0451735}{0.1} \approx 0.4187319 \]
Level 4 (Q9)

$L$ is the tangent to the curve $C : y = x^3 + 7$ at $x = 2$.

(a) Find the equation of the tangent $L$.
(b) Using the result of (a), find the area bounded by the $y$-axis, the tangent $L$ and the curve $C$.  

Level 4 (Q9)

Apply differentiation to find the equation of tangent

Integrate knowledge of differentiation and integration

Demonstrate sound knowledge and understanding of the calculus concepts

The area of triangle is incorrect

\[ y = \frac{x^2 + 7}{3} \]

When \( x = 2 \),
\[ y = \frac{2^2 + 7}{3} = \frac{15}{3} = 5 \]

\[ \frac{dy}{dx} \bigg|_{x=2} = 3(2) = 6 \]

Equation of \( L \):
\[ 12(x - 2) = (y - 15) \]
\[ 12x - 24 = y - 15 \]

\[ y = 12x - 9 \]

\[ y - 12x + 9 = 0 \]

For \( L \) when \( y = 0 \), \( x = \frac{3}{4} \), \( x, y = -7 \)

The required area:
\[ \int (\frac{1}{4}x^2 + 7) - \frac{1}{2} (12x - 9) + (\frac{3}{4}(x - 1)) \frac{x}{2} \]
\[ = \frac{1}{8} - \left( -\frac{15}{8} \right) + \frac{1}{8} \]
\[ = \frac{1}{2} \approx 12 \]

\[ = \frac{3}{16} \text{ unit}^2 \]
Level 3 (Q6)

Let \( u = \sqrt{\frac{2x+3}{(x+1)(x+2)}} \), where \( x > -1 \).

(a) Use logarithmic differentiation to express \( \frac{du}{dx} \) in terms of \( u \) and \( x \).

(b) Suppose \( u = 3^y \), express \( \frac{dy}{dx} \) in terms of \( x \).
Demonstrate adequate knowledge and understanding of the calculus concepts.

Apply log differentiation and chain rule.

The derivative is incorrect.
Level 2 (Q14)
(Refer to Slide 21)

The s.d. of sample mean is incorrect

Apply normal probability in this simple case
Demonstrate basic knowledge and understanding of the statistics concepts

Apply binomial probability in this simple case

The numerator of the conditional probability is incorrect

\[
\begin{align*}
(\text{I1}) \quad P \left( \frac{8}{37} \right) \\
&= \binom{8}{2} (0.91)^2 (0.088)^6 + \binom{8}{1} (0.088) (0.91)^7 \\
&= 0.8707 - 0.86680623 \\
&= 0.8681 \\
(\text{II}) \quad P \left( \frac{C(n)}{10} \right) \\
&= \binom{\frac{C(n)}{10}}{\frac{8}{37}} (0.088)^6 (0.91)^7 + \binom{\frac{C(n)}{10}}{\frac{7}{37}} (0.088)^7 (0.91)^6 \\
&= 0.8681623 \\
&= 0.053238 + 0.00332738 \\
&= 0.0652
\end{align*}
\]
Level 1 (Q11)

The manager, Mary, of a theme park starts a promotion plan to increase the daily number of visits to the park. The rate of change of the daily number of visits to the park can be modelled by

\[
\frac{dN}{dt} = \frac{k(25-t)}{e^{0.04t} + 4t} \quad (t \geq 0)
\]

where \( N \) is the daily number of visits (in hundreds) recorded at the end of a day, \( t \) is the number of days elapsed since the start of the plan and \( k \) is a positive constant.

Mary finds that at the start of the plan, \( N = 10 \) and \( \frac{dN}{dt} = 50 \).

(a) (i) Let \( v = 1 + 4te^{-0.04t} \), find \( \frac{dv}{dt} \).

(ii) Find the value of \( k \), and hence express \( N \) in terms of \( t \).

(b) (i) When will the daily number of visits attain the greatest value?

(ii) Mary claims that there will be more than 50 hundred visits on a certain day after the start of the plan. Do you agree? Explain your answer.

(c) Mary’s supervisor believes that the daily number of visits to the park will return to the original one at the start of the plan after a long period of time. Do you agree? Explain your answer.

(Hint: \( \lim_{t \to \infty} te^{-0.04t} = 0 \).)
Level 1 (Q11)

Demonstrate elementary knowledge and understanding of the calculus concepts

The derivative is incorrect

Cannot use (a)(i) to find an appropriate substitution

\[
\frac{dV}{dt} = 4t e^{-0.004t} (0.004t + 1) e^{-0.004t} (t)
\]

\[
= -0.016t^2 + 0.004t - 0.004t
\]

\[
= e^{-0.004t} (0 - 0.004t)
\]

When \( t = 0 \), \( \frac{dV}{dt} = 50t \)

\[
50 = \frac{dV}{dt}
\]

\[
\frac{dN}{dt} = \frac{2(50 - t)}{e^{-0.004t} + t} = \frac{50 - 2t}{e^{0.004t} + t}
\]

\[
\frac{dN}{dt} = \frac{2(50 - t)}{e^{0.004t} + t} dt
\]

\[
N = \int \frac{2(50 - t)}{e^{0.004t} + t} dt
\]

\[
\frac{dN}{dt} = (50 - t) e^{0.004t} + 4t)
\]

\[
\frac{dN}{dt} = (50 - t) (e^{0.004t} + 4t)
\]

\[
= (e^{0.004t} + 4t)
\]

\[
< 0 \text{ when } t < 75 \text{ at } t < 25
\]

\[
\frac{dN}{dt} = 0
\]

\[
50 - 2.5 = 25
\]

It attains its greatest value at \( t = 25 \) days
Section A

2. A snowball in a shape of sphere is melting with its volume decreasing at a constant rate of $4 \text{ cm}^3 \text{s}^{-1}$. When its radius is 5 cm, find the rate of change of its radius.

4. Find \[ \int \left( x^2 - \frac{1}{x} \right)^4 \, dx \, . \]
8. (a) Using integration by parts, find \[ \int x \cos x \, dx. \]
(b) 

\[ \text{Figure 1} \]

The \textbf{inner surface} of a container is formed by revolving the curve \( y = -\cos x \) (for \( 0 \leq x \leq \pi \)) about the \( y \)-axis (see Figure 1). Find the capacity of the container.
Let $\overrightarrow{OA} = 4\mathbf{i} + 3\mathbf{j}$, $\overrightarrow{OB} = 3\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} + \mathbf{j} + 5\mathbf{k}$.

Figure 2 shows the parallelepiped $OADBEFCFG$ formed by $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$. 

9.
(a) Find the area of the parallelogram $OADB$.

(b) Find the volume of the parallelepiped $OADBECFG$.

(c) If $C'$ is a point different from $C$ such that the volume of the parallelepiped formed by $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC'}$ is the same as that of $OADBECFG$, find a possible vector of $\overrightarrow{OC'}$. 
12. Let \( f(x) = \frac{4}{x-1} - \frac{4}{x+1} - 1 \), where \( x \neq \pm 1 \).

(a) (i) Find the \( x\)- and \( y\)-intercept(s) of the graph of \( y = f(x) \).

(ii) Find \( f'(x) \) and prove that
\[
f''(x) = \frac{16(3x^2 + 1)}{(x-1)^3(x+1)^3}
\]
for \( x \neq \pm 1 \).

(iii) For the graph of \( y = f(x) \), find all the extreme points and show that there are no points of inflexion.
(b) Find all the asymptote(s) of the graph of $y = f(x)$.

(c) Sketch the graph of $y = f(x)$.

(d) Let $S$ be the area bounded by the graph of $y = f(x)$, the straight lines $x = 3$, $x = a$ $(a > 3)$ and $y = -1$. Find $S$ in terms of $a$. Deduce that $S < 4 \ln 2$. 
In Figure 3, \( \triangle ABC \) is an acute-angled triangle, where \( O \) and \( H \) are the circumcentre and orthocentre respectively. Let \( \overrightarrow{OA} = a \), \( \overrightarrow{OB} = b \), \( \overrightarrow{OC} = c \) and \( \overrightarrow{OH} = h \).
(a) Show that \((h - a) \parallel (b + c)\).

(b) Let \(h - a = t(b + c)\), where \(t\) is a non-zero constant.
Show that

(i) \(t(b + c) + a - b = s(c + a)\) for some scalar \(s\),

(ii) \((t - 1)(b - a) \cdot (c - a) = 0\).

(c) Express \(h\) in terms of \(a\), \(b\) and \(c\).
Draft Level Descriptors (M2)

The typical performance of candidates at this level:
Level 5
- demonstrate comprehensive knowledge and understanding of the algebra and calculus concepts in the curriculum by applying them successfully at a sophisticated level to a wide range of unfamiliar situations
- communicate and express views and arguments precisely and logically using mathematical language, notations, tables, diagrams and graphs
- provide complex mathematical proofs successfully in a logical, rigorous and concise manner
- integrate knowledge and skills from different areas of the curriculum in handling complex tasks using a variety of strategies
Level 4

- demonstrate sound knowledge and understanding of the algebra and calculus concepts in the curriculum by applying them successfully to unfamiliar situations
- communicate and express views and arguments accurately using mathematical language, notations, tables, diagrams and graphs
- provide mathematical proofs successfully in a logical manner
- integrate knowledge and skills from different areas of the curriculum in handling a range of tasks
Level 3

- demonstrate adequate knowledge and understanding of the algebra and calculus concepts in the curriculum by applying them successfully to familiar and some unfamiliar situations
- communicate and express views and arguments appropriately using mathematical language, notations, tables, diagrams and graphs
- employ appropriate strategies to provide mathematical proofs
- integrate knowledge and skills from different areas of the curriculum in handling mathematical tasks in explicit situations
- apply algebra and calculus techniques in solving simple problems involving mathematical contexts successfully
Level 2

- demonstrate basic knowledge and understanding of the algebra and calculus concepts in the curriculum by employing simple algorithms, formulas and procedures in performing routine tasks
- communicate and express relevant views using mathematical language, notations, tables, diagrams and graphs
- employ routine techniques to arrive at some preliminary results in providing mathematical proofs
- use differentiation and integration in handling straightforward and clearly defined problems
- perform matrices and vectors operations in simple situations and solves systems of linear equations
Level 1

- demonstrate elementary knowledge and understanding of the algebra and statistics concepts in the curriculum by performing straightforward procedures according to direct instructions
- communicate and express simple ideas using mathematical language, notations, tables, diagrams and graphs
- find derivatives and integrals of simple functions
- perform straightforward matrix and vector manipulations
Annotated Examples (M2)
Level 5 (Q12)
(Refer to Slide 44)

(a) \( f(x) = x^3 - 4x^2 - 1 \)
- Intercept \( f(0) = -1 \)
- At \( y = 0 \),
  \[ f'(x) = 3x^2 - 8x \]
  \[ f'(x) = 0 \Rightarrow 3x^2 - 8x = 0 \]
  \[ x = 0, \quad x = \frac{8}{3} \]

\[ g(x) = \frac{1}{x^2(x-1)} \]
\[ g'(x) = \frac{-2}{x^3(x-1)^2} \]
\[ g''(x) = \frac{6}{x^2(x-1)^3} \]

The presentation of intercepts is incorrect

The second derivative is positive only on a certain range.
b) The vertical asymptote are \( y = x = 1 \) and \( x = -1 \) 

The horizontal asymptote is \( y = \lim_{x \to \pm \infty} f(x) = \lim_{x \to \pm \infty} \frac{a}{x^2 + 1} = 0 \)

\( \lim_{x \to -1} f(x) = \lim_{x \to -1} \frac{a}{x^2 + 1} = -1 \)

The horizontal asymptote is \( y = -1 \)

The graphs of the curve and asymptotes are correctly sketched.
Demonstrate comprehensive knowledge and understanding of the algebra and calculus concepts.

The limit notation is not very appropriate.

Can integrate knowledge from different areas of curriculum.
Level 4 (Q12)
(Refer to Slide 44)

The presentation of intercepts is incorrect

Complete the proof successfully
The graphs of the curve and asymptotes are correctly sketched.
Demonstrate sound knowledge and understanding of the algebra and calculus concepts.
Level 3 (Q11)

Let \( A = \begin{pmatrix} 2 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} \), \( P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \) and \( D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \).

(a) Let \( I \) and \( O \) be the \( 3 \times 3 \) identity matrix and zero matrix respectively.

(i) Prove that \( P^3 - 2P^2 - P + I = O \).

(ii) Using the result of (i), or otherwise, find \( P^{-1} \).

(b) (i) Prove that \( D = P^{-1}AP \).

(ii) Prove that \( D \) and \( A \) are non-singular.

(iii) Find \( (D^{-1})^{100} \).

Hence, or otherwise, find \( (A^{-1})^{100} \).
Apply matrix techniques in simple parts
Employ appropriate strategies, although not flawless, to provide mathematical proofs.

Demonstrate adequate knowledge and understanding of the algebra and calculus concepts.
Level 2 (Q7)

Solve the system of linear equations

\[
\begin{align*}
 x + 7y - 6z &= -4 \\
 3x - 4y + 7z &= 13 \\
 4x + 3y + z &= 9
\end{align*}
\]
Level 2 (Q7)

Apply Gaussian elimination to solve system of linear equations

Wrongly introduce two parameters

Demonstrate basic knowledge and understanding of the algebra and calculus concepts
Level 1 (Q3)
The slope at any point \((x, y)\) of a curve is given by \(\frac{dy}{dx} = 2x \ln(x^2 + 1)\).

It is given that the curve passes through the point \((0, 1)\).

Find the equation of the curve.

Wrongly apply integration by parts rather than substitution

Demonstrate elementary knowledge and understanding of the algebra and calculus concepts

The equation of the curve is \(y = 1\)
Thank you