本評卷參考乃香港考試及評核局專為本科練習卷而編寫，供教師參考之用。教師應提醒學生，不應將評卷參考視爲標準答案，硬背死記，活剝生吞。這種學習態度，既無助學生改善學習，學懂應對及解難，亦有違考試着重理解能力與運用技巧之旨。因此，本局籲請各位教師通力合作，堅守上述原則。

This marking scheme has been prepared by the Hong Kong Examinations and Assessment Authority for teachers’ reference. Teachers should remind their students NOT to regard this marking scheme as a set of model answers. Our examinations emphasise the testing of understanding, the practical application of knowledge and the use of processing skills. Hence the use of model answers, or anything else which encourages rote memorisation, will not help students to improve their learning nor develop their abilities in addressing and solving problems. The Authority is counting on the co-operation of teachers in this regard.
Hong Kong Diploma of Secondary Education Examination
Mathematics Compulsory Part Paper 1

General Marking Instructions

1. This marking scheme is the preliminary version before the normal standardisation process and some revisions may be necessary after actual samples of performance have been collected and scrutinised by the HKEAA. Teachers are strongly advised to conduct their own internal standardisation procedures before applying the marking schemes. After standardisation, teachers should adhere to the marking scheme to ensure a uniform standard of marking within the school.

2. It is very important that all teachers should adhere as closely as possible to the marking scheme. In many cases, however, students will have obtained a correct answer by an alternative method not specified in the marking scheme. In general, a correct answer merits all the marks allocated to that part, unless a particular method has been specified in the question. Teachers should be patient in marking alternative solutions not specified in the marking scheme.

3. In the marking scheme, marks are classified into the following three categories:
   - ‘M’ marks awarded for correct methods being used;
   - ‘A’ marks awarded for the accuracy of the answers;
   - Marks without ‘M’ or ‘A’ awarded for correctly completing a proof or arriving at an answer given in a question.

   In a question consisting of several parts each depending on the previous parts, ‘M’ marks should be awarded to steps or methods correctly deduced from previous answers, even if these answers are erroneous. However, ‘A’ marks for the corresponding answers should NOT be awarded (unless otherwise specified).

4. For the convenience of teachers, the marking scheme was written as detailed as possible. However, it is still likely that students would not present their solution in the same explicit manner, e.g. some steps would either be omitted or stated implicitly. In such cases, teachers should exercise their discretion in marking students’ work. In general, marks for a certain step should be awarded if students’ solution indicated that the relevant concept/technique had been used.

5. Use of notation different from those in the marking scheme should not be penalized.

6. In marking students’ work, the benefit of doubt should be given in the students’ favour.

7. Marks may be deducted for wrong units (u) or poor presentation (pp).
   a. The symbol $u-1$ should be used to denote 1 mark deducted for $u$. At most deduct 1 mark for $u$ in each of Section A(1) and Section A(2). Do not deduct any marks for $u$ in Section B.
   b. The symbol $pp-1$ should be used to denote 1 mark deducted for $pp$. At most deduct 1 mark for $pp$ in each of Section A(1) and Section A(2). Do not deduct any marks for $pp$ in Section B.
   c. At most deduct 1 mark in each of Section A(1) and Section A(2).
   d. In any case, do not deduct any marks in those steps where students could not score any marks.

8. In the marking scheme, ‘r.t.’ stands for ‘accepting answers which can be rounded off to’ and ‘f.t.’ stands for ‘follow through’. Steps which can be skipped are shaded whereas alternative answers are enclosed with rectangles. All fractional answers must be simplified.
<table>
<thead>
<tr>
<th>Solution</th>
</tr>
</thead>
</table>
| 1. \[
\frac{(m^5 n^{-2})^6}{m^4 n^{-3}} = \frac{m^{30} n^{-12}}{m^4 n^{-3} n^{12}} = \frac{m^{30-4}}{n^{3-3}} = \frac{m^{26}}{n^0} \]
| Marks | 1M |
| Remarks | for \((ab)^p = a^p b^p\) or \((a^p)^q = a^{pq}\) |

<table>
<thead>
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</table>
| 2. \[
\frac{5 + b}{1 - a} = 3b
\]
| Marks | 1M |
| Remarks | for \(3b(1 - a)\) |

<table>
<thead>
<tr>
<th>Solution</th>
</tr>
</thead>
</table>
| 3. (a) \[
9x^2 - 42xy + 49y^2 = (3x - 7y)^2 \]
| Marks | 1A |
| Remarks | or equivalent |

<table>
<thead>
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</table>
| (b) \[
9x^2 - 42xy + 49y^2 - 6x + 14y = (3x - 7y)^2 - 6x + 14y = (3x - 7y)^2 - 2(3x - 7y) = (3x - 7y)(3x - 7y - 2) \]
| Marks | 1A |
| Remarks | or equivalent |

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PP-DSE-MATH-CP 1–3
4. Let $x$ be the marked price of the chair.

\[ x (1 - 20\%) = 360 (1 + 30\%) \]

\[ x = \frac{360 (1.3)}{0.8} \]

\[ x = 585 \]

Thus, the marked price of the chair is $585.

<table>
<thead>
<tr>
<th>Solution</th>
<th>Marks</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x (1 - 20%) = 360 (1 + 30%)$</td>
<td>1M+1M+1A</td>
<td>pp–1 for undefined symbol</td>
</tr>
<tr>
<td>$x = \frac{360 (1.3)}{0.8}$</td>
<td>1M for $x (1 - 20%)$ + 1M for $360 (1 + 30%)$</td>
<td></td>
</tr>
<tr>
<td>$x = 585$</td>
<td>1A</td>
<td>$u$–1 for missing unit</td>
</tr>
</tbody>
</table>

\[
\text{The marked price of the chair} = \frac{360 (1 + 30\%)}{1 - 20\%} = 585
\]

----------(4)

5. Let $x$ litres and $y$ litres be the capacities of a bottle and a cup respectively.

\[
\begin{cases}
    \frac{x}{3} = 4 \\
    7x + 9y = 11
\end{cases}
\]

So, we have $7x + 9\left(\frac{3x}{4}\right) = 11$.

Solving, we have $x = \frac{4}{5}$.

Thus, the capacity of a bottle is $\frac{4}{5}$ litre.

<table>
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<tr>
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<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{x}{3} = 4$</td>
<td>1A+1A</td>
<td>pp–1 for undefined symbol</td>
</tr>
<tr>
<td>$7x + 9\left(\frac{3x}{4}\right) = 11$</td>
<td>1M</td>
<td>for getting a linear equation in $x$ or $y$ only</td>
</tr>
<tr>
<td>$7x + 9y = 11$</td>
<td>$u$–1 for missing unit</td>
<td></td>
</tr>
</tbody>
</table>

Let $x$ litres be the capacity of a bottle.

\[ 7x + 9\left(\frac{3x}{4}\right) = 11 \]

Solving, we have $x = \frac{4}{5}$.

Thus, the capacity of a bottle is $\frac{4}{5}$ litre.

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<tr>
<td>$7x + 9\left(\frac{3x}{4}\right) = 11$</td>
<td>1A+1M+1A</td>
<td>pp–1 for undefined symbol</td>
</tr>
<tr>
<td>$y = \frac{3x}{4}$ + 1M for $7x + 9y = 11$</td>
<td>1A for $y = \frac{3x}{4}$</td>
<td></td>
</tr>
<tr>
<td>$x = \frac{4}{5}$</td>
<td>1A</td>
<td>0.8</td>
</tr>
</tbody>
</table>

$u$–1 for missing unit

----------(4)
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>6. (a) ( \angle AOC )</td>
<td>1M</td>
<td>for considering ( \angle AOC )</td>
</tr>
<tr>
<td>( = 337° - 157° )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 180° )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thus, ( A, O ) and ( C ) are collinear.</td>
<td>1A</td>
<td>f.t.</td>
</tr>
<tr>
<td>(b) Note that ( BO \perp AC ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>The area of ( \triangle ABC )</td>
<td>1M</td>
<td></td>
</tr>
<tr>
<td>( = \frac{1}{2} (13 + 15)(14) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 196 )</td>
<td>1A</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>---------(4)</td>
</tr>
<tr>
<td>7. Note that ( \angle BCD = 90° ).</td>
<td>1A</td>
<td></td>
</tr>
<tr>
<td>Also note that ( \angle CBD = 180° - 90° - 36° = 54° ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Further note that ( \angle BAC = \angle BDC = 36° ).</td>
<td>1M</td>
<td></td>
</tr>
<tr>
<td>Since ( AB = AC ), we have ( \angle ACB = \angle ABC ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>So, we have ( \angle ABC = \frac{180° - 36°}{2} ).</td>
<td>1M</td>
<td></td>
</tr>
<tr>
<td>Therefore, we have ( \angle ABC = 72° ).</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle ABD )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = \angle ABC - \angle CBD )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 72° - 54° )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 18° )</td>
<td>1A</td>
<td>u-1 for missing unit</td>
</tr>
<tr>
<td>Note that ( \angle BAC = \angle BDC = 36° ).</td>
<td>1M</td>
<td></td>
</tr>
<tr>
<td>Since ( AB = AC ), we have ( \angle ACB = \angle ABC ).</td>
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<tr>
<td>So, we have ( \angle ACB = \frac{180° - 36°}{2} ).</td>
<td>1M</td>
<td></td>
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<tr>
<td>Therefore, we have ( \angle ACB = 72° ).</td>
<td></td>
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<tr>
<td>Also note that ( \angle BCD = 90° ).</td>
<td>1A</td>
<td></td>
</tr>
<tr>
<td>( \angle ACD )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 90° - 72° )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 18° )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \angle ABD )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = \angle ACD )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 18° )</td>
<td>1A</td>
<td>u-1 for missing unit</td>
</tr>
<tr>
<td></td>
<td></td>
<td>---------(4)</td>
</tr>
</tbody>
</table>
8. (a) The coordinates of \( A' \)
\[ (3, 4) \]
1A

(b) Let \((x, y)\) be the coordinates of \( P \).
\[
\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-(-2))^2}
\]

Thus, the required equation is \( x - 3y - 1 = 0 \).
1A

The coordinates of \( A' \)
\[ (3, 4) \]
1A

(b) Let \((x, y)\) be the coordinates of \( P \).
\[
\sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x-5)^2 + (y-(-2))^2}
\]

Thus, the required equation is \( x - 3y - 1 = 0 \).
1A

9. (a) The least possible value of the inter-quartile range of the distribution
\[
\frac{5 - 5}{2} \quad \text{or} \quad \frac{2 - 2}{2} = 0
\]
1M

The greatest possible value of the inter-quartile range of the distribution
\[
\frac{5 - 2}{2} = 3
\]
1A

(b) Since \( r = 9 \) and the median of the distribution is \( 3 \),
we have \( 9 + 8 > 12 + s \).
Therefore, we have \( s < 5 \).
So, we have \( s = 1, 2, 3 \) or \( 4 \).
Thus, there are 4 possible values of \( s \).
1A

---------(5)
10. (a) Note that when \( f(x) \) is divided by \( x-1 \), the remainder is 4.  
\[
f(x) = (x-1)(6x^2 + 17x - 2) + 4 \\
= 6x^3 + 11x^2 - 19x + 6
\]
\[
f(-3) = 6(-3)^3 + 11(-3)^2 - 19(-3) + 6 = 0
\]
(b) \( f(x) = (x+3)(6x^2 - 7x + 2) \\
= (x+3)(2x-1)(3x-2)
\]

11. (a) Let \( C = a + bx^2 \), where \( a \) and \( b \) are non-zero constants.  
So, we have \( a + (20^2)b = 42 \) and \( a + (120^2)b = 112 \).  
Solving, we have \( a = 40 \) and \( b = \frac{1}{200} \).  
The required cost  
\[
= 40 + \frac{1}{200}(50^2) \\
= $52.5
\]
(b) \( 40 + \frac{1}{200}x^2 = 58 \)
\[
x^2 = 3600 \\
x = 60
\]
Thus, the required length is 60 cm.
12. (a) The required duration
\[ \frac{63 - 32}{1} = 31 \text{ minutes} \]
\[ = 1M \quad 1A \quad \text{u-1 for missing unit} \]

(b) Suppose Ada and Billy meet at a place which is at a distance of \( x \) km from town \( P \).
\[ \frac{12}{78} = \frac{x}{120} \]
\[ x = 7.8 \]
Thus, Ada and Billy meet at a place which is at a distance of 7.8 km from town \( P \).
\[ = 1M+1A \quad 1M \text{ for ratio } 78:120 \]

(c) The average speed of Ada
\[ = \frac{12}{2} \]
\[ = 6 \text{ km/h} \]
The average speed of Billy
\[ = \frac{16 - 2}{2} \]
\[ = 7 \text{ km/h} \]
Note that \( 7 > 6 \).
Thus, Billy runs faster.
\[ = 1A \quad \text{f.t.} \]

During the period, Ada runs 12 km while Billy runs 14 km.
\[ = 1M \]
Note that 14 > 12.
So, the average speed of Billy is higher than that of Ada.
Thus, Billy runs faster.
\[ = 1A \quad \text{f.t.} \]
13. (a) Let \( n \) be the number of students in the group.
\[
\frac{6}{n} = \frac{3}{20} \\
n = 40
\]

\[k = 40 - 6 - 11 - 5 - 10 \]
\[= 8\]

(b) (i) The required angle
\[
\frac{5}{40} (360^\circ) \\
= 45^\circ
\]

(ii) Let \( m \) be the number of new students.
Assume that the angle of the sector representing that the most favourite fruit is orange will be doubled.
\[
\frac{5+m}{n+m} = \frac{(45)2}{360} \\
40 + m = 360 \\
20 + 4m = 40 + m \\
3m = 20
\]
Since 20 is not a multiple of 3, the angle of the sector representing that the most favourite fruit is orange will not be doubled.

---

pp–1 for undefined symbol

1M for considering \( \frac{5+m}{n+m} \)

1A f.t.

---
14. (a) \( \triangle BCD \sim \triangle OAD \)

(b) (i) Let \((0, h)\) be the coordinates of \(C\).

By (b), we have

\[
\left( \frac{12-h}{\sqrt{6^2+12^2}} \right)^2 = \frac{16}{45}.
\]

\[h^2 - 24h + 80 = 0\]

\[h = 4 \quad \text{or} \quad h = 20 \quad \text{(rejected)}\]

Thus, the coordinates of \(C\) are \((0, 4)\).

(ii) Note that \(AC\) is a diameter of the circle \(OABC\).

So, the coordinates of the centre of the circle are \((3, 2)\).

Also, the radius of the circle is \(\sqrt{(3-0)^2 + (4-2)^2} = \sqrt{13}\).

Thus, the equation of the circle \(OABC\) is \((x-3)^2 + (y-2)^2 = 13\).

Suppose that the equation of the circle \(OABC\) is

\[x^2 + y^2 + k_1x + k_2y + k_3 = 0, \quad \text{where} \quad k_1, k_2 \text{ and } k_3 \text{ are constants.}\]

\[
\begin{align*}
0^2 + 0^2 + k_1(0) + k_2(0) + k_3 &= 0 \\
6^2 + 0^2 + k_1(6) + k_2(0) + k_3 &= 0 \\
0^2 + 4^2 + k_1(0) + k_2(4) + k_3 &= 0
\end{align*}
\]

Solving, we have \(k_1 = -6\), \(k_2 = -4\) and \(k_3 = 0\).

Thus, the equation of the circle \(OABC\) is \(x^2 + y^2 - 6x - 4y = 0\).
15. (a) Let $s$ be the standard deviation of the scores in the test.

\[
\frac{36 - 48}{s} = -2 \\
\Rightarrow s = 6
\]

The standard score of John in the test

\[
\frac{66 - 48}{6} = 3
\]

(b) Note that the score of David is equal to the mean of the scores.

So, the mean of the scores remains unchanged.

The sum of squares of the deviations of the scores in the test remains unchanged while the number of students decreases by 1.

Therefore, the standard deviation increases.

Hence, the standard score of John decreases.

Thus, the standard score of John will change.
16. (a) The required probability
\[ \frac{C_{18}^{1}}{C_{4}^{}}, \]
\[ = \frac{68}{609}, \]
1M for numerator or denominator
\[ \text{Marks} \]
1A r. t. 0.112

The required probability
\[ = \frac{18}{30} \cdot \frac{17}{29} \cdot \frac{16}{28} \cdot \frac{15}{27}, \]
\[ = 68 \]
\[ = \frac{68}{609}, \]
1M for \( \binom{r-1}{n-1} \cdot \binom{r-2}{n-2} \cdot \binom{r-3}{n-3}, \ r < n \)
\[ \text{Marks} \]
1A r. t. 0.112

(b) The required probability
\[ = 1 - \frac{68}{609} \cdot \frac{C_{12}^{4}}{C_{4}^{}}, \]
\[ = \frac{530}{609}, \]
1M for \( 1 - (a) - p_{1} \)
\[ \text{Marks} \]
1A r. t. 0.870

The required probability
\[ = \frac{C_{18}^{1} + C_{2}^{1} + C_{3}^{1} + C_{4}^{1}}{C_{10}^{4}}, \]
\[ = \frac{530}{609}, \]
1M for considering 3 cases
\[ \text{Marks} \]
1A r. t. 0.870

The required probability
\[ = 1 - \frac{68}{609} \cdot \frac{12}{30} \cdot \frac{11}{29} \cdot \frac{10}{28} \cdot \frac{9}{27}, \]
\[ = \frac{530}{609}, \]
1M for \( 1 - (a) - p_{2} \)
\[ \text{Marks} \]
1A r. t. 0.870

The required probability
\[ = 4 \cdot \frac{18}{30} \cdot \frac{12}{29} \cdot \frac{11}{28} \cdot \frac{10}{27} + 6 \cdot \frac{18}{30} \cdot \frac{17}{29} \cdot \frac{12}{28} \cdot \frac{11}{27} + 4 \cdot \frac{18}{30} \cdot \frac{17}{29} \cdot \frac{16}{28} \cdot \frac{12}{27}, \]
\[ = \frac{530}{609}, \]
1M for considering 14 cases
\[ \text{Marks} \]
1A r. t. 0.870

\[ \text{----------(2)} \]
17. (a) \[
\frac{1}{1 + 2i} = \frac{1}{1 + 2i} \left( \frac{1 - 2i}{1 - 2i} \right) = \frac{1 - 2i}{5} = \frac{2}{5}i
\]

(b) (i) Note that \(\frac{10}{1 + 2i} = 2 - 4i\) and \(\frac{10}{1 - 2i} = 2 + 4i\).

The sum of roots
\[
\frac{10}{1 + 2i} + \frac{10}{1 - 2i} = (2 - 4i) + (2 + 4i) = 4
\]

The product of roots
\[
\left( \frac{10}{1 + 2i} \right) \left( \frac{10}{1 - 2i} \right) = 20
\]
Thus, we have \(p = -4\) and \(q = 20\).

(ii) When the equation \(x^2 - 4x + 20 = r\) has real roots, we have \(\Delta \geq 0\).

So, we have \((-4)^2 - 4(20 - r) \geq 0\).

Thus, we have \(r \geq 16\).
18. (a) By cosine formula,
\[ AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos \angle ACB \]
\[ AB^2 = 20^2 + 12^2 - 2(20)(12)\cos60^\circ \]
\[ AB = 4\sqrt{19} \text{ cm} \]

(b) By sine formula,
\[ \frac{\sin \angle BAC}{BC} = \frac{\sin \angle ACB}{AB} \]
\[ \frac{\sin \angle BAC}{12} = \frac{\sin 60^\circ}{4\sqrt{19}} \]
\[ \angle BAC = 36.58677555^\circ \]

Let \( Q \) be the foot of the perpendicular from \( C \) to \( AB \).
\[ \sin \angle BAC = \frac{CQ}{AC} \]
\[ CQ = 20\sin 36.58677555^\circ \]
\[ CQ = 11.92079121 \text{ cm} \]

Since \( \triangle ABC \cong \triangle ABD \), the required angle is \( \angle CQD \).
\[ \sin \frac{\angle CQD}{2} = \frac{1}{2} \frac{CD}{CQ} \]
\[ \sin \frac{\angle CQD}{2} = 0.587209345 \]
\[ \angle CQD = 71.91844786^\circ \]
\[ \angle CQD = 71.9^\circ \]

Thus, the angle between the plane \( ABC \) and the plane \( ABD \) is \( 71.9^\circ \).
By sine formula,
\[ \sin \angle BAC = \sin \angle ACB \]
\[ \frac{BC}{AB} \]
\[ \sin \angle BAC = \sin 60^\circ \]
\[ = \frac{12}{4\sqrt{19}} \]
\[ \angle BAC = 36.58677555^\circ \]
Let \( Q \) be the foot of the perpendicular from \( C \) to \( AB \).
\[ \sin \angle BAC = \frac{CQ}{AC} \]
\[ CQ = 20\sin 36.58677555^\circ \]
\[ CQ = 11.92079121 \text{ cm} \]
By symmetry, we have \( DQ = CQ \).
\[ DQ = 11.92079121 \text{ cm} \]
Since \( \triangle ABC \equiv \triangle ABD \), the required angle is \( \angle CQD \).
\[ CD^2 = CQ^2 + DQ^2 - 2(CQ)(DQ)\cos \angle CQD \]
\[ = 11.92079121^2 + 11.92079121^2 - 2(11.92079121)(11.92079121)\cos \angle CQD \]
\[ \angle CQD \approx 71.91844786^\circ \]
\[ \angle CQD \approx 71.9^\circ \]
Thus, the angle between the plane \( ABC \) and the plane \( ABD \) is \( 71.9^\circ \).

The area of \( \triangle ABC \)
\[ = \frac{1}{2}(AC)(BC)\sin \angle ACB \]
\[ = \frac{1}{2}(20)(12)\sin 60^\circ \]
\[ = 60\sqrt{3} \text{ cm}^2 \]
Let \( Q \) be the foot of the perpendicular from \( C \) to \( AB \).
\[ \frac{1}{2}(AB)(CQ) = 60\sqrt{3} \]
\[ \frac{1}{2}(4\sqrt{19})(CQ) = 60\sqrt{3} \]
\[ CQ = 11.92079121 \text{ cm} \]
Since \( \triangle ABC \equiv \triangle ABD \), the required angle is \( \angle CQD \).
\[ \sin \frac{\angle CQD}{2} = \frac{CQ}{CD} \]
\[ \sin \frac{\angle CQD}{2} = 0.587209345 \]
\[ \angle CQD \approx 71.91844786^\circ \]
\[ \angle CQD \approx 71.9^\circ \]
Thus, the angle between the plane \( ABC \) and the plane \( ABD \) is \( 71.9^\circ \).

(c) Let \( Q \) be the foot of the perpendicular from \( C \) to \( AB \).

Note that \( \sin \frac{\angle CPD}{2} = \frac{CD}{CP} \).

Since \( CP \geq CQ \), we have \( \angle CPD \leq \angle CQD \).

Thus, \( \angle CPD \) increases as \( P \) moves from \( A \) to \( Q \) and decreases as \( P \) moves from \( Q \) to \( B \).

---

PP-DSE-MATH-CP 1–15
19. (a) \(4000000(1 - r\%)^3 = 1048576\)

\[\begin{align*}
(1 - r\%)^3 &= \frac{1048576}{4000000} \\
1 - r\% &= 0.64 \\
r &= 36
\end{align*}\]

(b) (i) Let \(n\) be the number of years needed for the total revenue made by the firm to exceed \$9000000.

\[\begin{align*}
20000000 + 2000000(1 - 20\%)+ \cdots + 2000000(1 - 20\%)^{n-1} < 9000000
\end{align*}\]

\[\begin{align*}
2000000 \frac{(1 - (0.8)^n)}{1 - 0.8} > 9000000
\end{align*}\]

\((0.8)^n < 0.1\)

\[\begin{align*}
n \log 0.8 < \log 0.1
\end{align*}\]

\[\begin{align*}
n > \frac{\log 0.1}{\log 0.8}
\end{align*}\]

\[\begin{align*}
n > 10.31885116
\end{align*}\]

Thus, the least number of years needed is \(11\).

(ii) The total revenue made by the firm

\[\begin{align*}
20000000 &< 2000000 + 2000000(1 - 20\%) + 2000000(1 - 20\%)^2 + \cdots
\end{align*}\]

\[\begin{align*}
= \frac{2000000}{1 - 0.8} = 1000000
\end{align*}\]

Thus, the total revenue made by the firm will not exceed \$10000000.

(iii) The total revenue made by the firm minus the total amount of investment in the first \(m\) years

\[\begin{align*}
= \frac{20000000(1 - (0.8)^m)}{1 - 0.8} - \frac{4000000(1 - (0.64)^m)}{1 - 0.64}
\end{align*}\]

\[\begin{align*}
= \frac{10000000}{9} \left( 1 - (0.8)^m \right) - \frac{10}{9} \left( 1 - (0.64)^m \right)
\end{align*}\]

\[\begin{align*}
= \frac{10000000}{9} \left( 10(0.8)^m - 9(0.8)^m - 1 \right)
\end{align*}\]

\[\begin{align*}
= \frac{10000000}{9} \left( 10(0.8)^m + i \right) \left( 0.8)^m - 1 \right)
\end{align*}\]

Note that \((0.8)^m > 0\) and \((0.8)^m < 1\) for any positive integer \(m\).

Therefore, we have \(10(0.8)^m + 1 > 0\) and \((0.8)^m - 1 < 0\).

So, we have

\[\begin{align*}
\frac{20000000(1 - (0.8)^m)}{1 - 0.8} - \frac{4000000(1 - (0.64)^m)}{1 - 0.64} < 0
\end{align*}\]

Thus, the claim is disagreed.
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